The Solar parallax with Gaia

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Abstract

This document presents the principles of the determination of the size of the solar system in kilometres based on Gaia observations of (433) Eros in February 2019 and (52768) 1998 OR2 during its Earth's flyby of April 2020. The historical context is given to introduce the issue in a wider perspective and explain the value of such an exploitation of Gaia data for training and science education. Data analyses with different levels of modelling are discussed together with their results. Although there is no truly novel science to expect from Gaia in this area, it happens that thanks to the 1.5 million km baseline, the processing is relatively straightforward and despite the lack of refinement it ends up with the most accurate determination of the Earth-Sun distance with this important, although now outdated, technique.

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Contents

Terminological issue

Since 2012 (IAU 2012, Resolution B2) the Astronomical Unit (au) is a defining constant of the astronomical system of units. This means that it is a conventional length given as a pure number, as au $= 149\,597\,870.700$ km, without uncertainty. As a consequence the Astronomical Unit is no longer the subject of improvement from observations, while attempting to measure the size of the orbit of the Earth expressed in *au* still makes sense.

Given this context this note deals with an application of peculiar Gaia observations for the purpose of education, general public information, as though we were doing astronomy in the old way, trying to find the scale of the solar system directly from the solar parallax. The expression *astronomical unit* or its shortened notation AU that is employed in this text should not be taken at face value with their modern meaning. These are just convenient ways to express a scale factor, very close to the mean Earth-Sun distance, and perfectly in line with the historical usage. The context is always clear and a special notation AU is used sometimes when it is deemed necessary to distinguish from the official unit noted *au* or au. Hence AU stands for the estimate of the mean-distance to the Sun from the observations, while *au* is used for any distance expressed in astronomical units.

1 Introduction

Assessing the exact dimension of the solar system was referred to as *the noblest problem in astronomy* by G. B. Airy, the Astronomer Royal from 1835 to 1881. It remained a central issue in astronomy until very recently and now has shifted to that of the mass of the planets and the Sun, since the unit of length is fixed conventionally. Its importance for solar physics is obvious to understand the precise amount of radiation that flows out of the Sun or to get the actual size of the Sun, but farther away the distance to the Sun is also the yardstick from which the whole Universe is measured.

Considerable efforts have been exercised over the centuries to achieve this goal and astronomers have shown extraordinary inventiveness and zeal to exploit every celestial opportunity that presented to them, as challenging as they may be, and history shows that real challenges were faced.

The astronomical methods were primarily based on angular measurements, the classical tool of precision astronomy. But indirect means were used as well, like the dynamical properties of the motion of the Moon that are linked to the distance of the Sun, the time the light takes to cross the Earth's orbit concluded from the advanced or delayed eclipses of the satellites of Jupiter leads to the diameter of the Earth's orbit. Finally the combination of the velocity of light and the astronomical aberration, which involves the mean orbital velocity of the Earth, and then the size of its orbit.

By far the most historically significant are the angular sights of a source from two locations widely separated at the surface of the Earth (see Sec. 3). The minute defect of parallelism between the two lines of sight is in direct relation with the distance of the body and with the length of the line, a chord of the Earth, between the two observers. Both parameters were not within the control of astronomers: they have to patiently wait for the passage of a solar system body as close as possible to the Earth, like a very favorable opposition of Mars or Venus, or the coming of a new minor planet crossing the orbit of Mars and approaching the Earth at a fraction of the distance of the Sun. As for the baseline, the limitation was even more uncompromising: it would (unfortunately) never be larger than the Earth diameter.

In terms of science, Gaia can do nothing really new in this respect, and in any case nothing that really matters as far as science results are concerned, and nowhere in the Gaia science case one would find an allusion to a possible achievement in this field. Therefore the title of this note may seem really at odds with this statement, as Gaia and solar parallax paths should never crossed each other.

However we had in 2019, and above all in 2020, two passages of the minor planets (433) Eros and (52768) 1998-OR2 that came at relatively short distance from the Earth, 0.25 *au* for Eros in 2019, and 0.04 *au* for 1998-OR2. What made these events attractive for the solar parallax with Gaia is the particular location of the spacecraft: orbiting around the Sun-Earth L2, it offers a vantage point located at 1.5 million km from the Earth, 117 times its diameter. We have there a baseline that astronomers of even the last century, who tried to observe Eros from as many places they could, could not imagine as something conceivable.

So with Gaia at L2 and observers on the Earth, both observing Eros or 1998-OR2 almost simultaneously, nature offers us an unique opportunity to recast with a remarkably favorable geometry an old-principle that has been so important for the development of modern astronomy, and which happens to be similar to the very principle Gaia uses to establish the true scale of the stellar system, well beyond the outer border of the solar system. Instead of dealing with an angle of $1/100$ of a degree, which required the equipment and the skills of trained professional astronomers and meticulous data analysis, we have now an angle of several degrees, making the analysis within reach of almost everyone interested, with simple geometry and without the dozens of minute corrections that pervade any accurate astrometric reduction.

The education value of astronomy is no more to be demonstrated and the success of planetarium and public outdoor skywatching is a vivid proof of the pubic interest. With these specific observations of Gaia, one can go further and provide teachers and students with a set of real data to exercice their skills and understand what accuracy means and how hard it is to get it. They can learn the difficult trade of data processing, apply statistics and modelling, do numerical calculations with many significant figures and at the end have a real result that would have amazed astronomers just two generations before us.

2 The solar parallax: historical overview

The absolute length of the astronomical unit was derived from dedicated observations as an angular quantity called the solar parallax whose meaning is shown in Fig. 1. The measurement of the angle $\varpi_\odot$ being equivalent to the Sun distance expressed with the Earth's radius. The latter being known in metric unit, the procedure would yield the size of the solar system in the same unit. With the Kepler law of motion, a single measurement of a distance of one solar system body suffices in establishing the absolute scale of the solar system. The very first significant step in this direction was achieved in 1672 during a most favourable (small distance) opposition of Mars, making its parallax as large as possible. On the 04 September 1672, Mars approached the Earth at 0.381 *au*, very close to the smallest possible distance of 0.372 *au*. Observing from two distant points J.D Cassini in Paris and J. Richer in Cayenne found an equatorial parallax of $10''$, or

$$\frac{R_\oplus}{1\ \mathrm{au}} = 10'' \tag{1}$$

giving, $1\mathrm{au} \approx 1.32 \times 10^8$ km. This is too small by $\approx 10\%$, but for the first time astronomers had a sensible estimate of the real size of the solar system from a method whose principle was sound and could not be disputed.

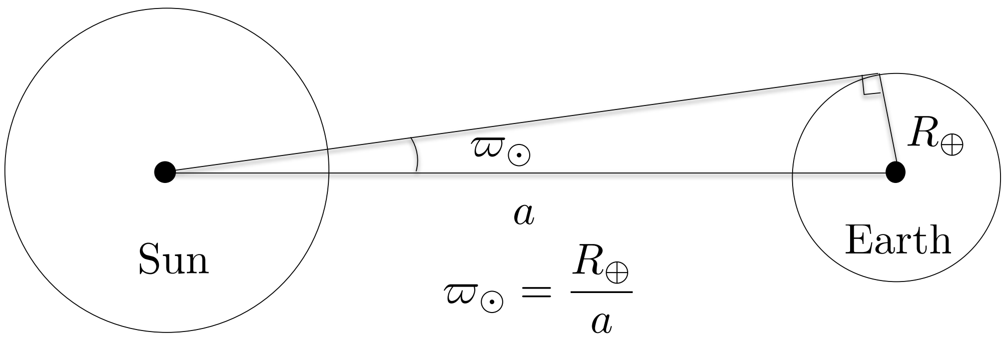

FIGURE 1: Definition of the solar parallax, as the angle subtended by the Earth's radius when seen from the Sun mean distance.

The rare transits of Venus across the solar disk offered another way of ascertaining the *au* [1] as noted first by the Scottish astronomer J. Gregory in 1663 and widely heralded by E. Halley

[1]The astronomical unit should be abbreviated as *au* since 2012 as stated in the recommendation of the International Astronomical Union in its Resolution B2. This is also the notation given by the BIPM in its official list of secondary units. It is still common to find instead AU or ua.

in 1716. The advantage of Halley's proposal is still extant since he proposed to replace pure angular measurements by timings of the moment the dark disk of the planet is seen encroaching on the bright solar disk. Given the angular speed of Venus relative to the Sun it is easy to show that a better accuracy can in principle be reached with the timing than with classical position sights. Halley claimed that the transit duration could be assessed to few seconds of time and consequently the distance to the Sun to one part to few thousandths, given a difference of duration of up to 20 mn between locations on the Earth. International cooperations were put in place for every occurrence of the Venus transit in 1761, 1769, 1874, 1882 to observe and time the passages from the most remote places on the Earth. This led to adventurous expeditions that have been reported in many books and most is available on-line or in popular accounts Woolf (1959), Maor (2000).

Regarding the astronomical aim, the results were not on a par with the expectations and never reached the accuracy claimed by the illustrious astronomer. The extensive discussion of the four transits by S. Newcomb in 1892 ended up with a solar parallax of $\varpi_\odot = 8\overset{''}{.}79 \pm 0\overset{''}{.}018$ (current determination $8.794\,143\cdots''$) or a value for the Sun-Earth distance of $(149.7 \pm 0.3) \times 10^6$ km. After so much trouble and cost, this was in some sense a very unsatisfactory situation in relation to the achievements of the planetary theories at the same time and after the triumph of the solar system dynamics with the discovery of Neptune in 1846.

A fortunate circumstance cast some lights in a gloomy landscape with the discovery in 1898 of the minor planet Eros (433 Eros) simultaneously at Berlin and Nice, the first of the near-Earth objects to be identified. Eros comes within the orbit of Mars and favourable oppositions that repeat at interval of 30-40 years may bring the planet to less than $0.2au$ from the Earth, closer than any other solar system object known at that time. The first such passage took place in 1901 and the next good one was due in 1931. Again a broad international cooperation was set up to observe and reduce the observations and led to a solar parallax of $\varpi_\odot = 8\overset{''}{.}790 \pm 0\overset{''}{.}002$. It was the most accurately known value for the solar distance at that time, and this value has remained the standard until mid-1960 when radar measurements, a new technique, gave a more accurate value for the distance to the Sun.

Although indirect methods, not based on direct angular measurement of the parallax, were applied to find the true size of the solar system, we restrict here to angular astrometry, since this is the kind of data that Gaia provides.

3 Triangulation in the solar system

3.1 Principle of angular triangulation

The principle put into its simplest form is just a trigonometric triangulation and has been known for centuries, and applied by Hipparchus to the Moon, before it was effectively used for the solar

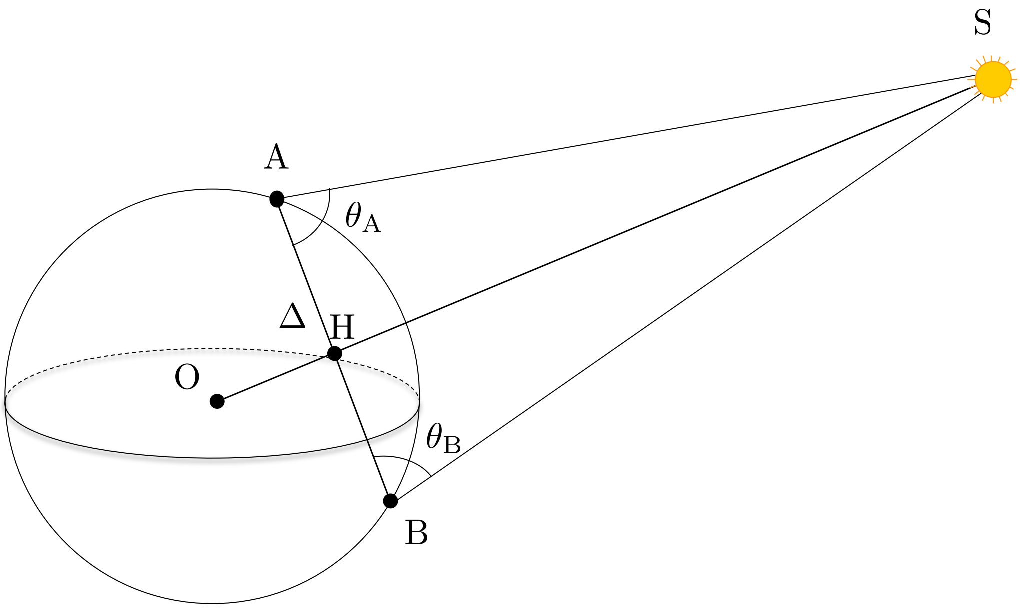

FIGURE 2: Overall principle of the triangulation from the sights of a celestial body from two widely separated locations on the Earth. For the Sun the two directions AS and BS are almost parallel, within less than $10''$.

parallax. A seen in Fig. 1, the solar parallax, and more exactly the mean equatorial horizontal parallax of the Sun, is the angle subtended by the radius of the Earth as seen from the Sun, when the Earth is at its mean distance. This angle is related to the distance Earth-Sun by,

$$\sin \varpi_{\odot} = \frac{R_{\oplus}}{a}$$

and given its smallness ($< 10''$), this is equivalent to,

$$\varpi_{\odot} = \frac{R_{\oplus}}{a} \tag{2}$$

Now for the measurement, one considers two observers situated at two widely well positioned locations on Earth who would measure accurately the angle between the line joining them (the baseline of the triangle) and the two directions towards the Sun, as illustrated in Fig. 2. It is clear that once the length of the baseline Δ, the two angles θ_A, θ_B are known, the distance OS can be evaluated by solving a triangle. For the Sun, one has always $\theta_A + \theta_B \simeq \pi$ and $OS \simeq HS$ without loss of accuracy.

While the principle is extremely straightforward, simple geometry, elementary underlying mathematics, its application to the Sun is extremely hard: the two directions AS and BS are nearly parallel. This means that not only must the two angles be assessed with great accuracy, but, and this is more difficult, the reference direction (here the chord AB) must be accessible at both places. Even with great optical sights, the sources of systematic errors are numerous and limit the final accuracy attainable.

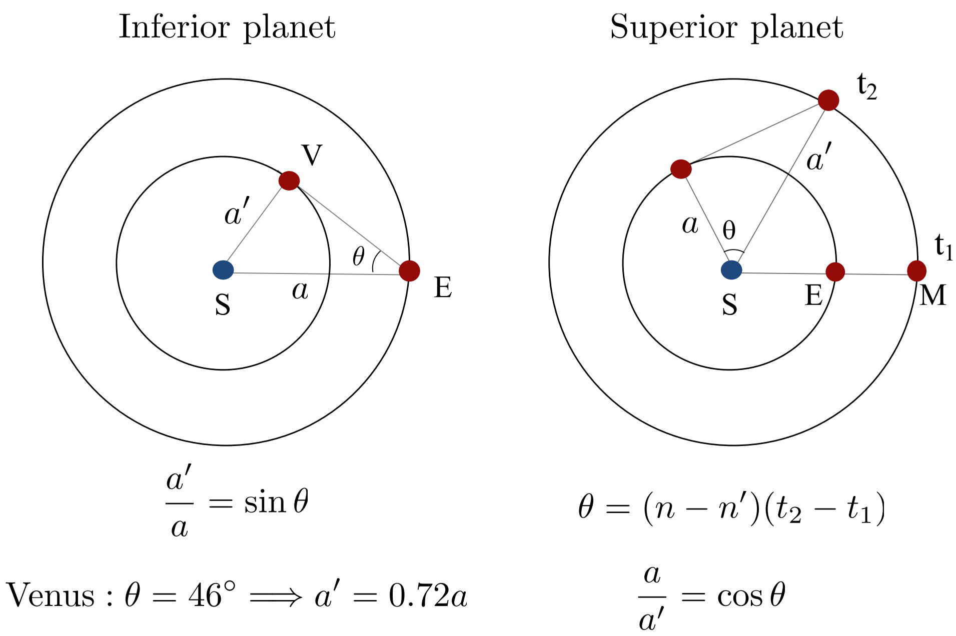

FIGURE 3: Determination of the relative distances in the solar system without the 3rd Kepler law. Left: for an inner planet, right : for an outer planet.

3.2 Relative and absolute size of the solar system

As mentioned earlier, measuring the solar parallax yields the distance to the Sun in Earth radius, and ultimately in common units since the size of the Earth was known to a better accuracy than that achievable for the solar parallax. But we will see below that the measurements were done on Mars, Venus or few minor planets instead of the Sun, taking advantage of the fact that the relative scale of the solar system was known (how far is Saturn compared to Mars for example) and just one distance was enough to have the true scale of the whole system.

It is commonly said that this fortunate circumstance rests upon the 3rd law of Kepler, relating the orbital period to the size of the orbit. This is true, but historically even before Kepler it was possible to place the planets in their relative position in a heliocentric model, and Copernicus knew that Jupiter was typically five times farther from the Sun than the Earth. The principles are shown in Fig. 3, for an inner planet (left panel) and superior planet (right panel).

At the moment Venus reaches it largest elongation (angular distance to the Sun), we have a right triangle at V and then a'/a is given by $\sin\theta$. Therefore, we know its distance to the Sun relative to that of the Earth. For an outer planet, this is not so direct, but remains easy. Starting from the opposition, when the Sun, Earth and the outer planet are aligned in this order, one waits until the outer planet reaches its quadrature. At that time we have a right triangle at the Earth, and from the time elapsed since the opposition one knows the difference of heliocentric longitude between the two bodies. Therefore a'/a is known and so are the relative proportions of the outer solar system.

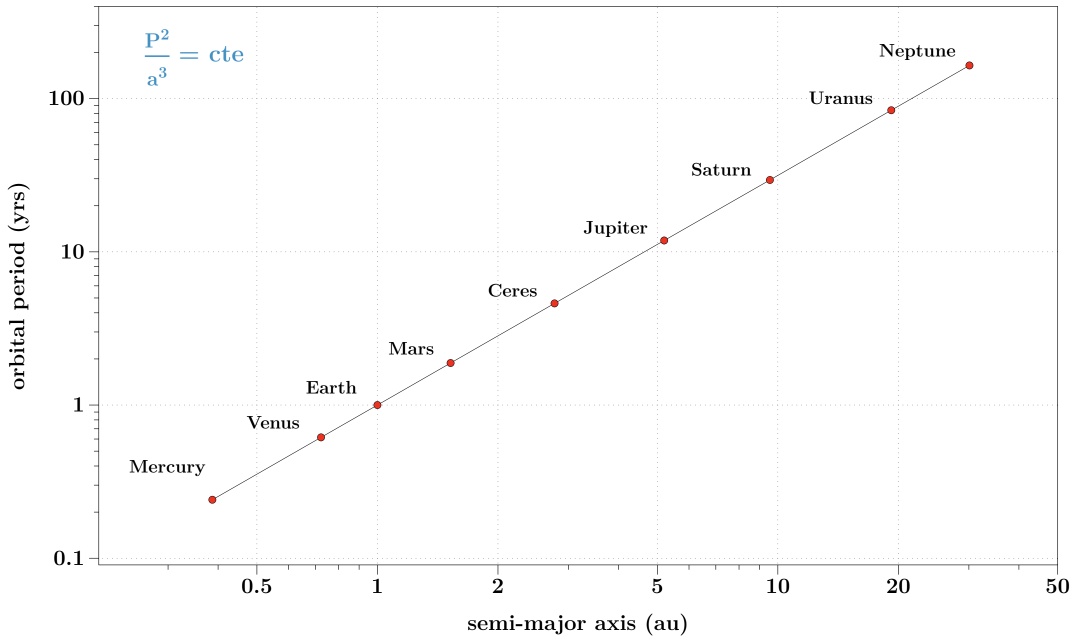

FIGURE 4: Graphical representation of the 3rd law of planetary motion. The relative scale of the solar system is fully determined by the orbital periods. A single distance measurement to a solar system body is enough to know at once all the other distances.

Obviously the use of the 3rd Kepler law (see Fig. 4), simplifies considerably the problem and allows one to deal with non-circular orbits. From this one concludes that measuring the true distance, at any time, of a single body orbiting the Sun, is enough to have the distances of all the other bodies. Thanks to that property we speak of establishing the solar parallax from observations, even though something different is actually measured with the parallactic angle of Mars, Venus or even a minor planet. The standard value for the Sun-Earth pair, can be deduced from a particular parallax from the scale model of the solar system.

4 Some outstanding results

4.1 Opposition of Mars in 1672

The first serious attempt to apply the triangulation principle to the Sun took place during a very favorable opposition of Mars in 1672 (Fig. 5), few years after the onset of the Académie Royale des Sciences in France and the creation of an observatory in Paris, and three years before the erection of the Greenwich Observatory. J.D. Cassini was heading the Paris institution while J. Flamsteed was appointed in 1675 as astronomer in charge at Greenwich. Both prepared themselves to observe Mars for the purpose of ascertaining the solar parallax.

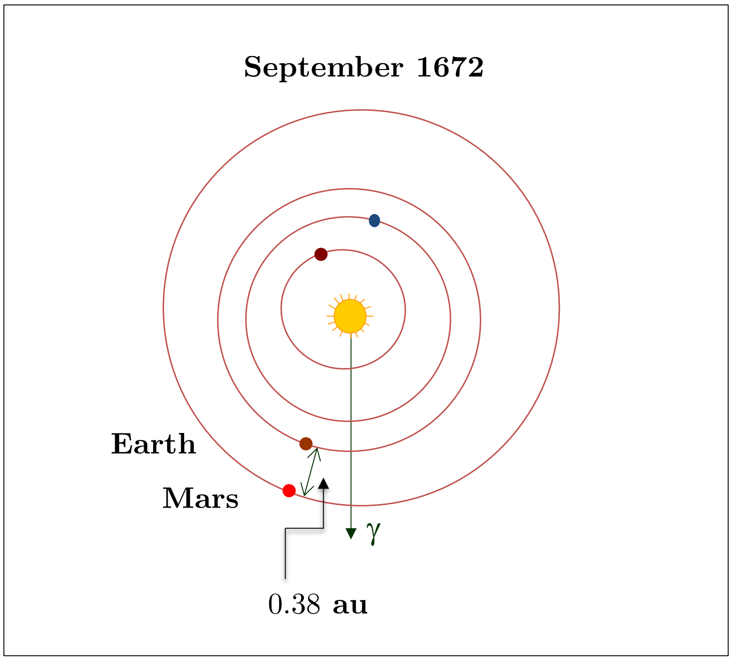

FIGURE 5: Configuration of the inner planets during the 1672 opposition of Mars. The smallest distance achievable between the Earth and Mars is 0.372 *au*.

As Mars was coming at 0.381 *au* from the Earth, the parallactic angle to estimate was the largest one could hope to have at this time. Knowing the distance of Mars leads to that of the Sun, or stated in an other way, the distance between the Earth and Mars was known in *au*, and obtaining that distance in a metric unit resulted in the *au* in the same metric unit. The circumstances are sketched out in Fig. 5, with a distance of only 0.381 *au*, very close to the smallest possible distance (Mars came to 0.3727 *au* in 2003). The French apply the textbook triangulation with two observers, and near simultaneous observations (not really achievable given the longitude difference and the absence of direct communication). J. Richer was sent to Cayenne, in today French Guiana, and Cassini observed from Paris. The planet was referred to a set of nearby bright stars as shown in Fig. 6.

Flamsteed, was alone, and observing from a single station, he took telescopic sights at the beginning and end of the night with an interval of 6 hours. In the meantime he was transported by the rotation of the Earth at several thousands km and it was as though he had observed from two places, but at different times. During the interval Mars has moved against the stars, but this motion over few hours could be allowed for in the analysis with the computed geocentric motion. Cassini calculated a solar parallax of $9.5''$ and commented that *it was almost impossible to be certain of 2 or 3 seconds in the total parallax of Mars*, while Flamsteed measurements ended up with of solar parallax rounded to $10''$, or a distance around 130 million kilometres, much larger than the traditional estimates inheritated from the tradition. This was a great success and a vindication of the superiority of the telescope and the wire micrometer for accurate sights,

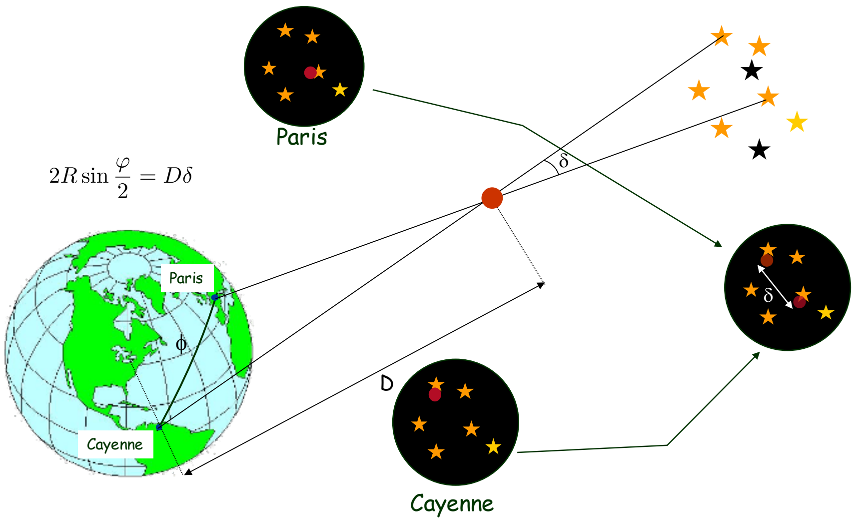

FIGURE 6: Sketch of the triangulation of Mars with simultaneous observations from two widely separated stations. Mars parallax is seen against the background stellar field as viewed in Paris and Cayenne.

compared to naked-eye observations (see van Helden (1986)).

The principle of triangulation of the solar system was in place and all the subsequent favorable oppositions of Mars were used to improve the result, albeit with moderate success. Then Halley came advocating the advantage that could be drawn from the transits of Venus to gauge the solar system. Although this was still a measurement of parallax, the angular sights were replaced by timings, and this method does fit naturally into this presentation, concerned with the direct application of angular measurements. I don't go into the details, but results are shown in Table 2 and I skip directly to the minor planets.

4.2 Solar parallax with minor planets

Besides Mars, the use of minor planets at opposition has been sporadic during the second half of the XIXth century. The German astronomer J.G. Galle, the who found Neptune in 1846 at Leverrier's request, came with the proposal in 1872 of using the upcoming favorable oppositions of (25) Phoceae and then (8) Flora for the purpose of measuring the distance of the Sun (Galle (1873a), Galle (1873b)). Minor planets may not be as close to the Earth as Mars at opposition (0.8 *au* in these instances instead of 0.38 for Mars), but they have a starlike appearance, making the astrometric sights easier and more accurate than with the disk of Mars. From observations against background stars at twelve observatories, Galle deduced a parallax of $8.87''$.

Later the method using the diurnal method (same as Flamsteed with Mars), was applied to the planet (3) Juno by J. Lindsay and the Cape astronomer Sir David Gill. More systematic operations were set up for the opposition of (7) Iris, (12) Victoria and (80) Sappho with the par-

COMBINATION OF RESULTS AND GENERAL DISCUSSION.

Collecting the definitive results from the preceding investigations of the value of the Mean Solar Parallax, we have—

	π_0	Probable Error.	Combining Weight.
From *Victoria*, Vol. I., Part 2, p. 532	8″·8013	± 0″·0061	2·69
„ *Sappho* „ „ 3, „ 83	·7981	± ·0114	0·77
„ *Iris* „ „ 4, „ 168	·8120	± ·0090	1·23

The combining weights are the reciprocals of the squares of the probable errors (unity $= \pm 0''\cdot 01$). Having regard to these weights, we have, in the mean

$$\pi_0 = 8''\cdot 8036 \text{ with weight } 4\cdot 69,$$

and the probable error $\pm 0''\cdot 0046$

which is the definitive value of the Solar Parallax as derived from the heliometer observations of *Victoria*, *Sappho*, and *Iris* under discussion, provided that there are no systematic errors peculiar to any of the series which have not yet been discussed.

FIGURE 7: Conclusions of the coordinated observations of Iris, Sappho and Victoria at the end of three years of data reduction by Sir David Gill at the Cape Observatory.

ticipation of observatories of both hemispheres. Nearly 30 observing places were cooperating with heliometers and meridian refractors, collecting voluminous data over almost one year. The processing took three more years until Gill published the result in 1897, (Gill, 1897),

$$8.802'' \pm 0.005''$$

or equivalently,

$$\text{au} = 149\,465\,000\text{km}$$

> *a value that can be regarded as absolutely reliable within such limits as indicated by the value of its probable* error.

In Fig. 7 showing his final conclusion, one sees also that he is aware that the probable errors ignore the unknown systematical effects that could be larger. This was motivated by the fact that determinations with the constant of aberration, the parallax of Mars or the gravitational methods were not always in agreement with the boundaries of the probable errors.

4.3 The oppositions of Eros

The unexpected discovery of (433) Eros brought the principle to more promising footings. The minor planet (433) Eros was discovered twice on the night of 13 August 1898 by the German

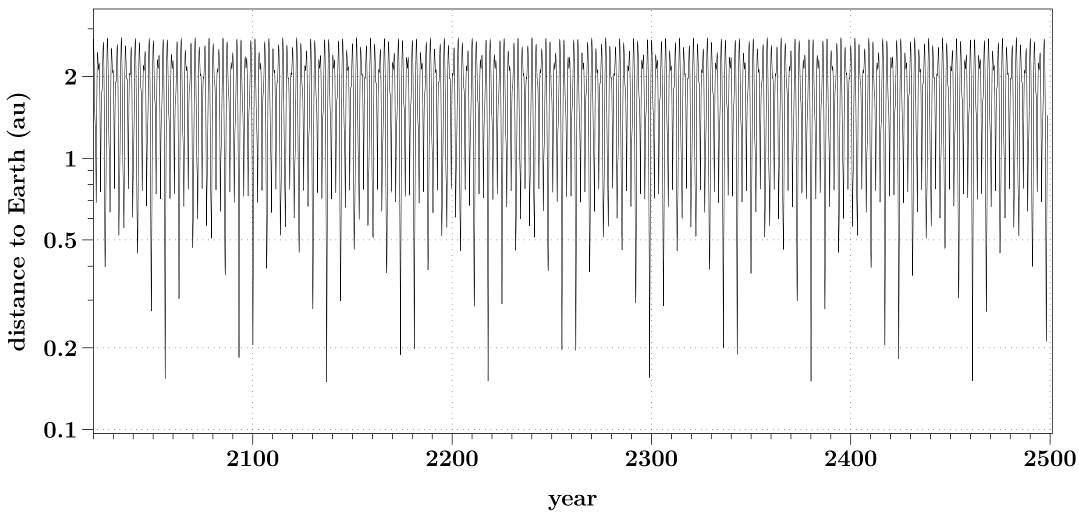

FIGURE 8: Eros: Distance to the Earth between 2020 and 2500. The smallest distance very close to 0.15 au recurs every 80 years with two less pronounced approaches in the middle of the interval.

astronomer Carl Gustav Witt at the Berlin Observatory and the French astronomer Auguste Charlois at the Nice observatory (but he did not communicate his finding). One realised quickly that its orbit did not lie entirely between Mars and Jupiter, as its perihelion comes within the orbit of Mars and in fact the planet spent most of its time within the orbit of Mars. During its orbital revolution Eros could come closer to the Earth than any other known solar system body, with the exception of the Moon, and seemed to be the best candidate to extend the search of the scale of the solar system.

With a semi-major axis $a = 1.458$ *au* and an eccentricity $e = 0.223$, the aphelion $a(1-e) = 1.133$ *au* is just outside the Earth orbit. Given the orbital inclination, at the most favorable oppositions the distance could be as low as 0.15 *au*, that made it a target of choice to determine the solar parallax. Eros was the first near-Earth object found, and was followed up to now by several thousands. This is the second brightest in this category.

The favorable oppositions, near perihelion implying a small distance to the Earth, recur at intervals of about 40 years as one can see in Fig. 8, with the next very close approach due to in 2055.

Over a shorter time scale, covering the discovery epoch and the present time, we have a similar diagram in Fig. 9, showing that the first close approach of 1901 followed the discovery by just 2 years, which put the astronomical community in great hurry to put an international collaboration in place. The procedures with photographic plates had to be defined in great detail and the observatories selected. Given the means of communication, it is even amzing that this was considered feasible within this timescale. But the expected reward was so promising, that the preparation moved forward quickly and successfully.

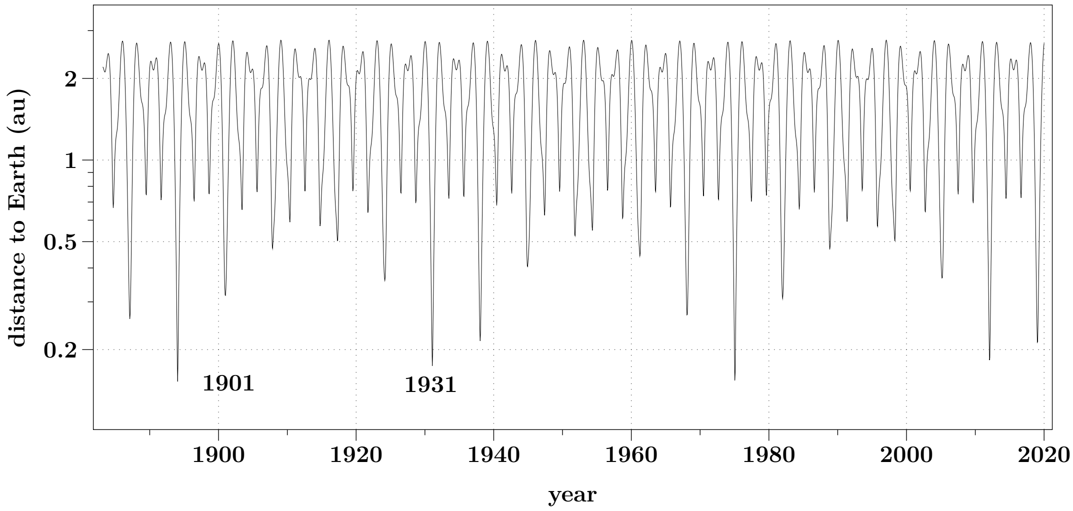

FIGURE 9: Eros : Distance to the Earth between 1890 and 2020, with the two historical passages of 1901 and 1931, respectively at 0.31 and 0.17 *au*

During the opposition of 1900–1901, a worldwide program was launched to coordinate parallax measurements of Eros to determine the solar parallax. More than 50 observatories of the northern hemisphere were engaged in this operation, under the supervision of the Cambridge astronomer A.R.Hinks who stated in a public lecture that (Hinks, 1905),

> *the problem of the determination of the distance of the Sun is, in some respects at least, the most fundamental of the whole range of astronomy, for the number it represents is involved in almost any calculation of distances and masses, of sizes and densities, either of planets or their satellites or stars.*

It took nearly ten years to carry out the data analysis and the results were published in 1910 (Hinks, 1910) as,

$$\varpi_\odot = 8.806'' \pm 0.004''$$

corresponding to a distance of 149 400, 000 km. This was compatible and with a similar accuracy as the investigations reported by Gill in 1897 on more distant minors planets. Clearly the method was better than with Mars, but above all far superior to the disappointing transits of Venus. One sees in Fig. 9 that another favorable opposition was due in early 1931 at a smaller distance of 0.17 *au*. A similar program of international cooperation was set up in less hurry, with observations extending over 1930–1931. This was led by Sir Harold Spencer-Jones, the Astronomer Royal, who also reduced the observations. Again, this was a 10-year labour, to gather the observations, sort them out according to the observational technique, discuss the errors and decide on the correct weighing. The result was epoch-making and the solar parallax which has resulted became the standard value for more than two decades as,

$$\varpi_\odot = 8.790'' \pm 0.001''$$

or

$$\text{AU} = 149\,670\,000 \text{ km}$$

It is worth quoting the confidence expressed by H. Spencer-Jones after this achievement:

> *Considering the extent of the collaboration, the large amount of observational data, the perfection of the methods used, the watchful elimination of the sources of errors, the careful discussion, we may say that another determination of the same or higher order of quality is not to be expected in the near future.*

He was essentially correct about the fact the method had come to an end, and only a new approach would arrive at a better value. We know today that the actual error is four times as large as his uncertainty, with the modern and conventional value of

$$\text{au} = 149\,597\,870.700 \text{ km}$$

or

$$\varpi_\odot = 8.794\,143\,836..''$$

The value of the Astronomical Unit (i.e the Earth-Sun distance) obtained by this program was considered definitive until around 1960, when radar and dynamical parallax methods started producing more precise measurements. On should note that the latter value deviates from the previous best more than could be expected from their quoted uncertainties. Therefore just before landing on the Moon, scientists did not know the exact size of the solar system to better than a few 10^{-4} in relative precision, and this after a quest extending over three centuries. It is akin today to the determination of the Newton gravitational constant, not much better known than that.

4.4 Synthesis to 1980

I have tried to summarise in a single table what I consider as being the knowledge of the scale of the solar system at different epochs. I have not collected all the results (this would fill a full volume) but my own appreciation of what was really known at a particular time. This is based on the scatter of individual results at each epoch, or the discussions of the results, the actual confidence people placed in the published results, or from the adoption by the community of a reference value used in the official publications. For the latter values it is clear that the choice made proceeded with the same philosophy: given the various results available at a time with non overlapping range of probable errors, what should we consider as the most reasonable distance

TABLE 2: Evolution of the knowledge of the size of the solar time with the time. Until 1960 the quantity measured is the solar parallax, then this is the distance with the velocity of light, and the last line gives the modern defining constant. The additional column is computed. The 1964 IAU value combines radar and optical angular measurements.

year	$\varpi_\odot$ $''$	AU 10^6 km	comment
1600	180	7.3	from Hipparchus
1700	9.0 – 10	130 – 145	Mars 1672
1800	8.5 – 9.1	145 – 155	Venus 1761-1769
1800	9.0	146	*Nautical Almanac* 1801-1833
1832	9.1	145	Mars
1835	8.57	153	Encke Venus 1761-1769
1866	8.90	147.8	*Nautical Almanac*
1890	8.79 – 8.88	148 – 149.6	Venus 1874-1882
1900	8.77 – 8.87	148.3 – 150	Mars, Flora
1900	8.848	148.7	*Nautical Almanac*
1910	8.802 – 8.810	149.30 – 149.46	Eros 1901
1940	8.789 – 8.791	149.65 – 149.68	Eros 1931
1950	8.80	149.5	IAU
1964	8.794	149.60	IAU 1968
1980	8.794 15	149.597 8	Radar 1960-1980
2000	8.794 143 837	149.597 870 69	Viking, Radar
2012	8.794 143 836...	149 597 870.700	IAU defining constant

to the Sun and with which level of confidence, normally reflected by the number of digits given in the parallax.

Even though, with the historical perspective, one can reasonably be disappointed by the results achieved with the Venus transits, at least regarding the distance of the Sun, the enthusiasm was not abated even in the mid-XIXth century, as is apparent from the emphatic quotation below of the noted historian of physical astronomy R. Grant in 1852:

> *When we consider the ingenuity of the method employed in arriving at this determination, and the refined nature of the process by which it is carried into effect, we cannot refrain from acknowledging it to be one of the noblest triumphs which the human mind has ever achieved in te study of physical science (Grant, 1852)*

However after the last two passages of Venus, considerable organisational efforts, terrible hardships for some in remote places and unbearable climates and vast expenses in many countries, the size of the solar system was not yet ascertained to better than 3% in relative accuracy or to within 5 millions kilometres in size. The situation improved, but by no more than an order of magnitude with the minor planets and the two successful Eros passages of 1901 and 1931 as discussed above.

4.5 Beyond the solar system

The extension of the triangulation method beyond the solar system is precisely the main tool Gaia applies to find the scale of the Universe. The yardstick is now the size of the orbit of the Earth and the targets are the stars instead of the planets. The principle is outlined in Fig. 10 and

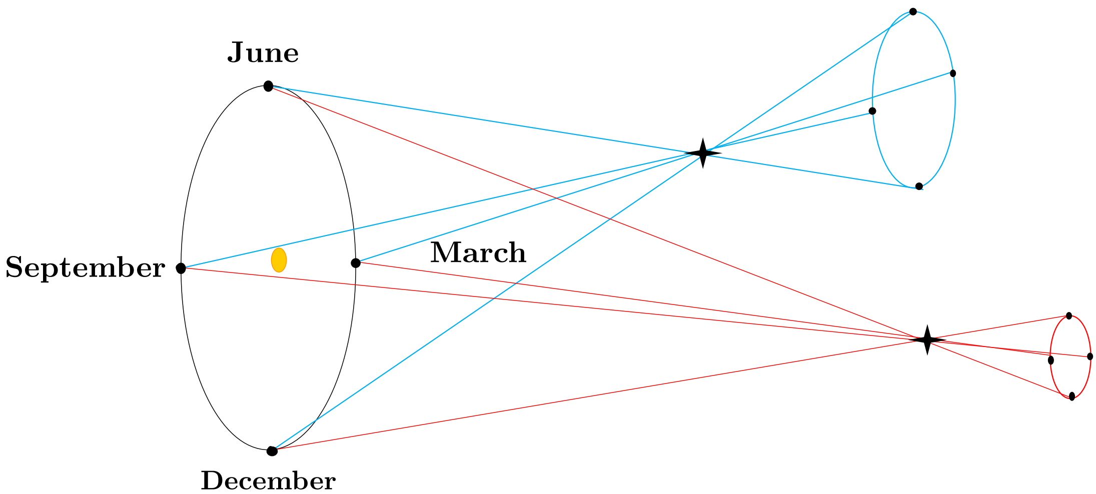

FIGURE 10: The annual parallactic motion of a nearby and a distant star. From repeated observations of the stars Gaia detects the annual apparent wobbling of the stars and determines the size of the parallactic ellipse, which is inversely proportional to their distances.

rests exactly on the triangulation method discussed above, but with a longer baseline needed to reach much larger distances. Provided the stars are not infinitely remote compared to the size of the Earth's orbit, our annual displacement translates into a reflex apparent displacement of the stars on the sky, since during the year the different lines joining the observer to the star are not parallel. The farther the star, the smaller the parallactic ellipse. More precisely its size is proportional to the reciprocal of the star distance. The parallax of a star is defined by the angle subtended at the star by one astronomical unit or half the apparent diameter of the Earth orbit when seen from the star. Mathematically one has for the parallax $\varpi_\star$ of a star at distance d from the solar system,

$$\varpi_\star = \frac{a}{d}$$

where $a = 1\,\mathrm{au}$. Therefore a single elementary geometric principle applies to different scales, and understanding how the solar system has been surveyed is a key to understand how Gaia probes the depth of the Milky Way.

5 Gaia and the solar parallax

5.1 How Gaia can contribute

Gaia, the ESA space astrometry mission, is primarily dedicated to an extremely accurate astrometry survey of the Galaxy, most of its targets being stars. Although several $100\,000$s minor planets are regularly observed with a remarquable wealth of science output, nothing is expected from Gaia in this area of *old fashioned astronomy*. This is all the more true as the astronomical unit by itself is no longer the subject of investigation since it has become a defined quantity, without error, directly in metric unit. Therefore the only serious science impact Gaia may have on this matter will be indirect, when the global solar system ephemeris (at JPL or at IMCCE) will make use of the astrometry of minor planets to fit their masses and have at the end a better set of fitted parameters, including the masses of the Sun and the planets. Then we will also have the distance of the Earth to the Sun expressed directly in km or in conventional *au*, but not exactly equal to 1 *au*.

However, if a minor planet is observed at close approach to the Earth by Gaia and simultaneously from the ground, then we can reproduce the old experiments on Mars or Eros, just to demonstrate how parallaxes are determined from angular sights of the same source from two different places. But here instead of having a baseline limited by the size of the Earth, we are fortunate to have one of the telescope located at 1.5 million kilometres from us, a unique baseline in the history of astronomy. Hence, the goal is not to produce a valuable science result, on a par with the best estimates of the Sun-Earth distance, but to provide the necessary material, explanations and algorithms with the simplest geometry as possible, without the intricacies needed to reach the best accuracy permitted by the observations. No one will spend as our glorious predecessors 10 years of labour to analyse the data and discuss every bit of the procedures.

By going too far and trying to fit the best solution, we will miss our goal to show the essential, the principles with a processing accessible to amateur scientists, students who want to learn and leave the utmost technical details to the specialists. Technicalities are important and absolutely necessary to get the best science out of observations but this is not what one aims at here. In addition this would be a waste of time, since no valuable science can be anticipated.

5.2 Principle

The possibility to use Gaia data to find the solar parallax implies that one has at least :

- Astrometric sights of one minor planet from Gaia;
- The planet is relatively close to the Earth during at this time, that is to say closer than 0.5 au, so that the parallax angle is at least $1°$;
- Ground based observations are available at the same time, of nearly the same time, with perfectly known locations (longitude, latitude, altitude) of the observing stations;
- the distance to Gaia is known in kilometres, without using the conversion factor between the au and the km ;
- The distance from the Earth to the planet, at each observation can be computed accurately in au, without using a conversion from km.

If we collect all these elements as in Fig. 19, we see that the geometry is elementary. The observations at Gaia and on the Earth give the angles G and E, and then P is known. Now with simple trigonometry one gets,

$$EP = EG\,\frac{\sin G}{\sin P} \tag{3}$$

giving EP in kilometres. As EP has been computed from the orbit of the planet in astronomical units, we end up with the AU in km. Here we have assumed that both observations are contemporaneous and that Earth based observation has been done at the center of the Earth. These two features are the main complications in the data analysis that we discuss below.

5.3 Accuracy achievable

Let's try now to estimate the accuracy that can be reached with this ideal configuration. Consider first that the only sources of uncertainties are the angular sights at G and at E. Then we will examine the others factors, like the length of the baseline and the computed distance to the planet.

One can use better notations to do the maths, with $EP = \rho$, $G = \gamma$, $E = \xi$, and $P = \varpi$ and for the baseline $EG = B$. The n,

$$\rho = B\,\frac{\sin\gamma}{\sin\varpi}$$

It is clear that one has essentially for the uncertainty propagation,

$$\frac{d\rho}{\rho} \approx \frac{d\varpi}{\tan\varpi} \tag{4}$$

and $d\varpi = d\gamma + d\xi$. The angle γ from Gaia observations has a $1-\sigma$ uncertainty of $0.06''$, which is the standard accuracy of the Gaia finder, which is the source of the astrometric data in this project. The uncertainty of ξ is that from the ground based data, and can be very variable from station to station, depending on the equipment, the skill of the observers, the seeing and the processing of the CCD frame. It can be as good as $0.10''$, but is more likely to be close to $0.5''$. But we will see that no ground based observation is really carried out at the same time as Gaia, and some pre-processing is need to estimate what would be the Earth observation at Gaia time by interpolating a set of actual observations. This amounts to computing a path of the planet on the sky within one or two days based on a set of observations. The final accuracy is hard to know precisely but should not be much worse than $0.5''$. Taking this value, and $\varpi \simeq 2$ to $10°$. Then one has potentially,

$$\frac{d\rho}{\rho} \quad \text{between} \quad 1.4\times10^{-5} - 7\times10^{-5} \tag{5}$$

This assumes that the angle ξ is taken from the centre of the Earth, which clearly is not true. Neglect the difference first, and add the error to the measurement uncertainty. The origin of the correction is shown in Fig. 20, where a planet at distance Δ from the center of the Earth is seen in the direction ζ_{A} from the location A at the surface of the Earth, and would be seen at the same time in the direction ζ_{O} from the center, with both directions being referred to the same vertical. In the cases of interest in this project, one has always $\Delta \gg R_\oplus$. Even at the closest approach of 1998-OR2, at 0.04 au one has $R_\oplus/\Delta < 0.001$ and the angle at P is always $< 0.06°$. One has,

$$\zeta_{\mathrm{A}} - \zeta_{\mathrm{O}} \approx \frac{R_\oplus}{\Delta}\sin\zeta_{\mathrm{A}} \tag{6}$$

and in practice one does not observe close to the horizon and it's reasonable to take as a good approximation,

$$\zeta_{\mathrm{A}} - \zeta_{\mathrm{O}} \approx 0.5\frac{R_\oplus}{\Delta} \tag{7}$$

This must be substitute in (4) for $d\varpi$,

$$\frac{d\rho}{\rho} \approx \frac{0.5R_{\oplus}/\Delta}{\tan\varpi} \tag{8}$$

but with $\tan\varpi \approx B/\Delta$ this gives,

$$\frac{d\rho}{\rho} \approx 0.5\frac{R_{\oplus}}{B} \approx 2\times 10^{-3} \tag{9}$$

which does not depend on the distance of the planet. This is an important property that tells us that by considering topocentric observations as though they were done at the centre of the Earth, therefore neglecting the topocentric shift, we can determine the solar parallax or the distance Earth-Sun with a relative accuracy of 2 parts in a thousand. The processing is much simpler and more accessible to groups with less expertise in astronomical calculations, and despite this drastic approximation, the result remains attractive. A quick look at table 2 is enough to convince everyone that a fractional accuracy of 0.002 from a single observation would have delighted astronomers of the first half of the last century.

5.4 Minor planets with Gaia

During its science mission Gaia is regularly observing about $300,000$ known asteroids, with typically 15 observations per year for the planets that remain brighter than $G = 20.7$ throughout their orbit. At the end and for each epoch a sub-mas position will be published, together with photometric data. The processing takes years, and between the observations and the publication of the results, several years go, (Gaia Collaboration et al., 2018).

However, at the detection time, within 24h of the on-board detection by the Gaia finder, a very crude position is computed using the first attitude solution and the on-board pixel location. The accuracy is reasonably constant at 60 mas for each coordinate. Not so bad, but far from the Gaia capabilities. At least this allows to check whether a particular object has been detected and have its apparent direction from Gaia.

In Fig. 9 we see that Eros visited us again at small distance in 2019, more precisely in mid January 2019. Gaia was already in its routine science program and could have caught the planet in its net. Given the typical sampling time of Gaia, this would be a bit lucky to have it during this short period of time. But we were fortunate, and there are two Gaia observations during this close approach, in February 2019, while the planet was at 0.24 *au* from us.

The geometric configuration on 13 February 2019 is shown close to scale in Fig. 11 with the respective distances. Although in the figure the parallactic angle of $\approx 2^{\circ}$ looks very small and makes the figure unpleasant and hardly usable, this is a very large angle from an astronomical point of view, when compared to the parallaxes of at most $50''$ of the historical measurements.

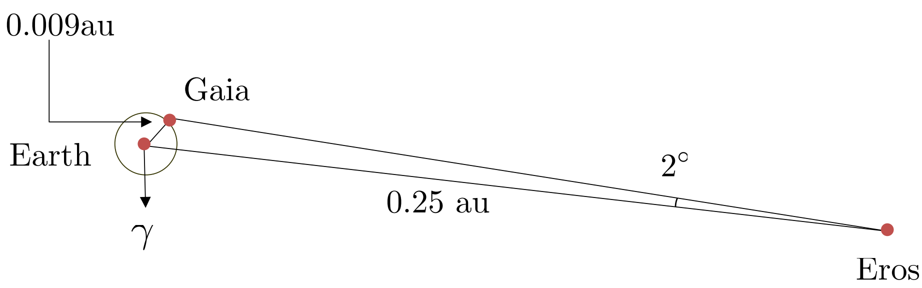

FIGURE 11: The geometric configuration between Gaia, the Earth and Eros during the Gaia observations on 13 February 2019.

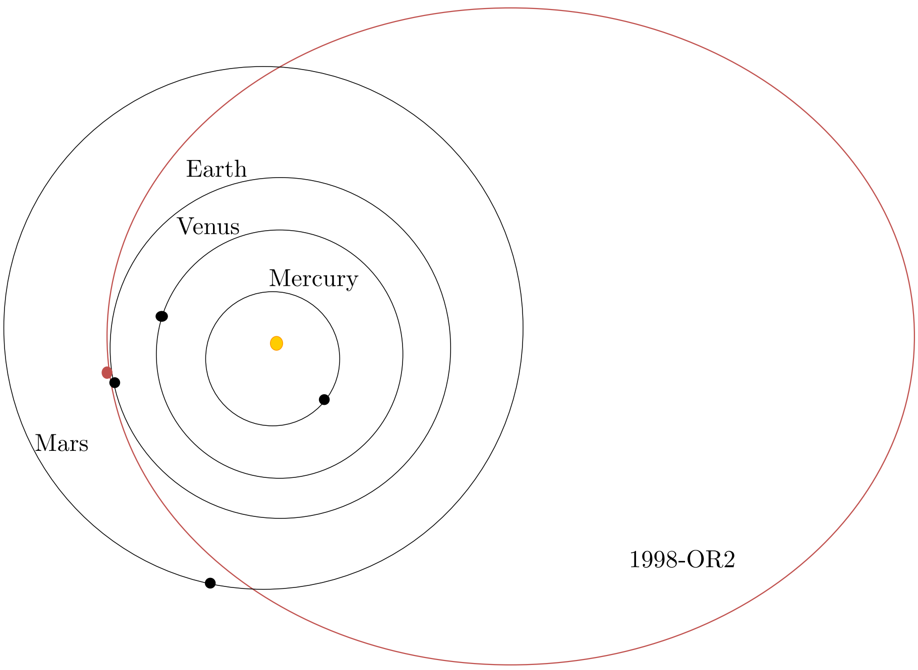

FIGURE 12: Orbital paths of the inner planets and of 1998OR2 with their respective positions during the fly-by of April 2020. The orbital inclinations are ignored in this plot.

But another planet happened to pass even closer in April 2020. The near-Earth object (52768) 1998 OR2 was discovered in July 1998 at the Haleakala Observatory. This is small body of about 2 km in diameter orbiting the Sun on a highly eccentric orbit ($e = 0.57$) with an orbital period of 3.7 years. With a perihelion distance of 1.018 *au* it can come very close to the Earth, as one can see in Fig. 12, although this is projected on the ecliptic and it does not show the effect of inclination. Fig. 13 gives the result of a numerical integration of its motion in the form of the distance to the Earth between 1950 and 2100. One sees the close approach of 2020, so close and quick that it is referred to as a fly-by, and a closer, non threatening passage, expected in 2079 at 0.012 *au*.

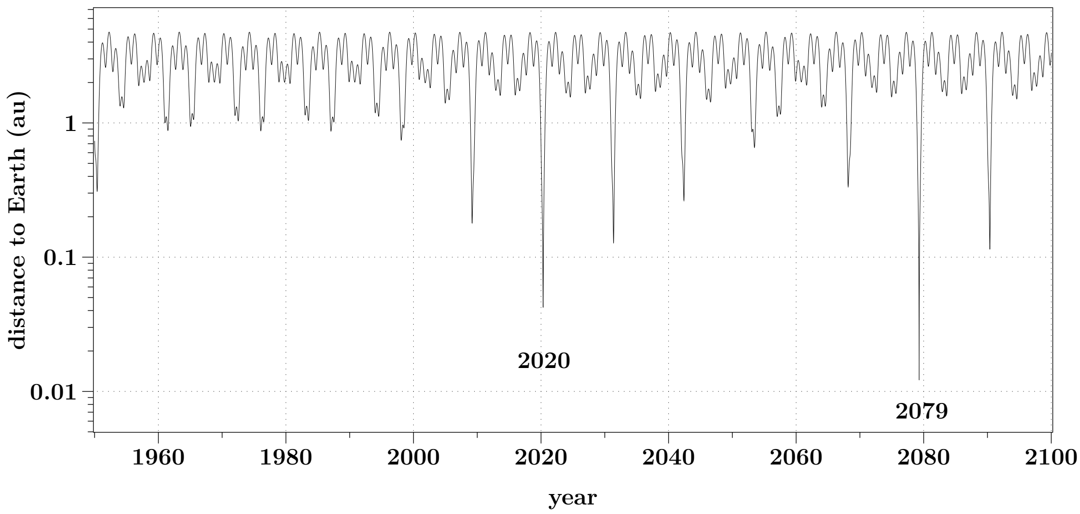

FIGURE 13: Distance of 1998-OR2 to the Earth between 1950 and 2100, with the two close approaches of 2020 at 0.042 *au* and 2079 at 0.012 *au*.

A closeup view in Fig. 14, shows the distance to the Earth and to Gaia during the fly-by. The distance to Gaia is even smaller than to the Earth at closest approach with a minimum distance of 0.038 *au* between Gaia and the planet. It is a stroke of luck that Gaia was pointing just in the right direction on the 29 April to observe twice the planet, while it was at 0.042 *au* from the Earth. We have also observations 10 days earlier at 0.075 *au* and in January 2020 at 0.22 *au*. In total these are eight observations of 1998-OR2 during the close approach of 2020.

The geometric configuration on 28 April is sketched in Fig. 15, with the respective distances to the Earth and Gaia and the parallactic angle of $12°$ at the planet. Obviously a parallactic angle of this magnitude has never been used in the past to measure the solar parallax, which results from both the large baseline (0.011 *au*) and the nearness of the planet.

5.5 The Gaia observations

The general conditions of the observable passages by Gaia are given in table 3, with the number of detected transits, the distance to the Earth and to Gaia and the parallactic angle at that time. This angle is very large for a solar system object compared to the maximum of $\sim 50''$ of the best passage of Eros in 1931, just because our baseline is 1.5×10^6 km instead of something comparable to the radius of the Earth. This is the single most important factor at the root of this outreach activity and its main driver.

The ten observations of Eros, in 2019, and 1998-OR2 in 2020, are given in table 4, as time, right

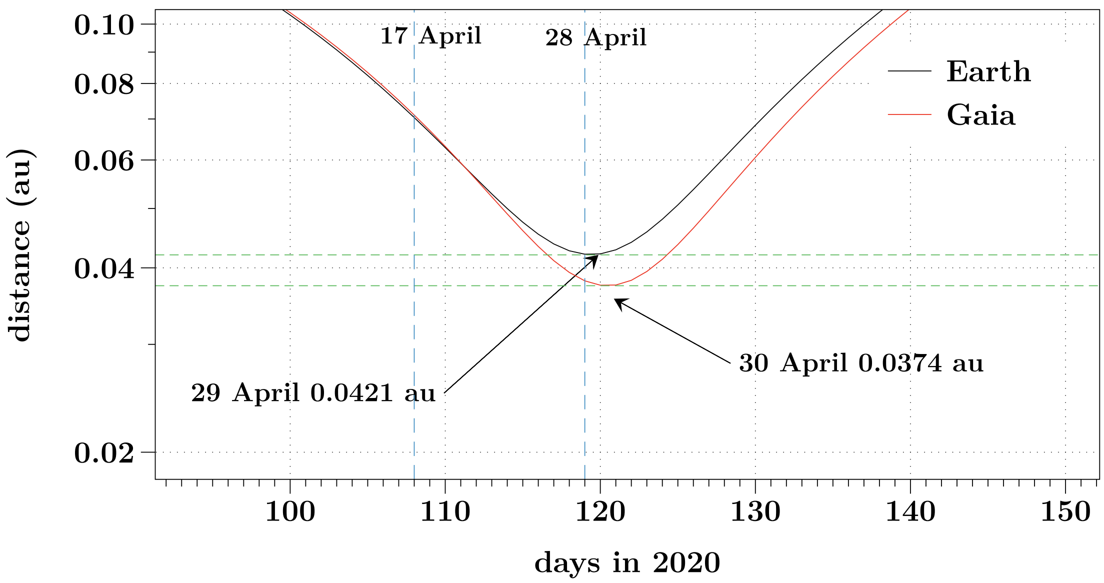

FIGURE 14: The fly-by of 1998-OR2 in April 2020, with the distance to the Earth and Gaia in *au*. Gaia observations took place on the 17 and 28 April.

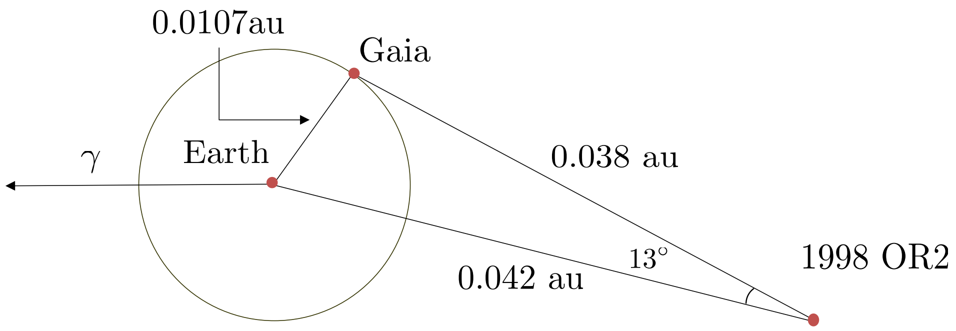

FIGURE 15: The geometric configuration between Gaia, the Earth and 1998-OR2 during the Gaia observations on 28 April 2020.

ascension and declination in the ICRF frame. The time found in Gaia data has been corrected to bring the Gaia timing, given at the last pixel at the AF1 chip, to the mid-crossing of SM where the detection was in fact done. The shift is -12.08 s in PFOV and -7.22 s in FFOV. The internal Gaia timing in TCB has been transformed to the TT scale to match the ground based data.

In table 5 we provide the computed geocentric position vector of Gaia in km, in cartesian and spherical coordinates at the same times, in the ICRF frame. It is important to stress that Gaia

TABLE 3: Passages of Eros and 1998-OR2 observed with Gaia in 2019 and 2020. Nobs is the number of detected transits with Gaia during the passage, $\Delta_{\rm E}$, $\Delta_{\rm G}$ the distances to the Earth and to Gaia. The last column is the parallactic angle.

planet	date	Nobs	$\Delta_{\rm E}$	$\Delta_{\rm G}$	ϖ
			au	au	deg
Eros	13 Feb 2019	2	0.247	0.242	1.4
1998 OR2	09 Mar 2020	3	0.228	0.225	2.3
1998 OR2	17 Apr 2020	3	0.072	0.073	8.5
1998 OR2	28 Apr 2020	2	0.042	0.038	13.5

position is given in km, and this does not result from some conversion using the *au* in km. Certainly a sub-km accuracy for the barycentric ephemeris would need a complete solar system ephemeris to process the tracking data. But for the geocentric position, and primarily the distance, the orbital tracking uses Doppler signal and direct range measurements, that are referred to Earth stations, well located, and everything comes out in km, never in astronomical units. In addition, there is also for Gaia, and this is the only modern spacecraft with that feature, a regular optical tracking giving the astronomical coordinates on the sky, with an accuracy of about 10 mas. Therefore one has the distance in km and the angular direction from astrometry. Therefore the baseline is known in metric unit and our approach is fair as far as one wants to find the astronomical unit in kilometres from astrometric data.

And finally in table 6 one finds the computed geocentric distances to Eros and 1998-OR2 at the observation times. This is computed in *au* from a numerical integration, starting with the orbital elements at a reference epoch. In this case, this is the opposite of the situation encountered with Gaia: the semi-major axis is naturally *au* and the integration can be conducted with an ephemeris, like VSOP, using only *au* and a scale factor for the gravitational forces based on the Gauss constant.

TABLE 4: The ten observations carried out by Gaia of Eros and 1998-OR2 in 2019 and 2020 during close approaches. Observation times have been transformed to the TT scale from the Gaia internal TCB for convenience and brought to the detection time from AF1 to SM. The column time is in days since J2010.0 and ra, dec are coordinates in the ICRF.

Planet	obs	date UTC	time TT	ra deg	dec deg
Eros	1	2019-02-13T12:27:42.4	3330.520041	84.294157	14.932476
	2	2019-02-13T14:14:17.1	3330.594054	84.346041	14.882177
1998 OR2	3	2020-03-09T11:25:14.6	3720.476664	113.573624	43.771898
	4	2020-03-09T13:11:53.6	3720.550726	113.553501	43.765217
	5	2020-03-09T17:25:28.3	3720.726823	113.506073	43.749073
1998 OR2	6	2020-04-17T08:42:15.2	3759.363476	121.466647	24.698680
	7	2020-04-17T12:55:48.5	3759.539557	121.639273	24.400109
	8	2020-04-17T14:42:26.8	3759.613611	121.712729	24.272702
1998 OR2	9	2020-04-28T08:47:40.0	3770.367236	141.963026	-14.817575
	10	2020-04-28T13:01:01.7	3770.543181	142.565622	-15.912499

TABLE 5: Position vector Earth-Gaia for the ten observations of table 4, in cartesian and spherical coordinates.

obs	x_G km	y_G km	z_G km	ρ_G km	α_G deg	δ_G deg
1	-1060167.1	799595.6	348671.7	1372909.1	142.9758	14.7123
2	-1061882.0	797487.3	348224.1	1372894.5	143.0930	14.6931
3	-1440513.8	-33036.2	80667.0	1443148.8	181.3138	3.2043
4	-1440840.1	-35389.8	79995.2	1443493.0	181.4070	3.1768
5	-1441596.7	-40987.3	78394.9	1444308.4	181.6286	3.1115
6	-1264067.9	-928802.4	-247846.7	1588071.0	216.3074	-8.9787
7	-1262332.4	-931208.3	-249219.0	1588314.2	216.4158	-9.0275
8	-1261603.9	-932214.4	-249794.9	1588416.1	216.4611	-9.0479
9	-1160648.8	-1059697.6	-333073.8	1606550.0	222.3968	-11.9655
10	-1159006.2	-1061619.8	-334489.6	1606927.2	222.4889	-12.0142

TABLE 6: Computed distances in astronomical units from the Earth to the observed planet for the ten observations of table 4.

obs	ρ_P *au*	obs	ρ_P *au*
1	0.24788363	6	0.07286612
2	0.24807434	7	0.07217205
3	0.22971115	8	0.07188101
4	0.22943454	9	0.04237553
5	0.22877679	10	0.04227522

5.6 Ground based observations

In the historical section of this presentation I have alluded to the large international cooperations that were put in place to coordinate observations at different places of the Earth around the passage of Eros near the Earth. The observation epochs were carefully selected and, when possible, attempts were exercised to have observations of Eros for example, with similar times at different places. In this exercice we deal with an opportunity rather than a careful planning, organised in advance of the events. The time sampling is set by Gaia scanning law and without any liberty. Our ten observations have been recorded by Gaia at specific times, and no observer on the ground was aware of the project and asked to point his/her telescope to Eros or 1998-OR2. So to get observations from the ground we have to rely on the existence of a large community of observers, particularly interested by the close approaches of Eros in 2019 or 1998 OR2 in 2020 and hope that some have recorded observations within ~ 1 day of Gaia. There are also several automatic stations or programs dedicated to monitor near-Earth asteroids that have collected data for these two planets.

Fortunately astronomers with the Minor Planet Center (MPC) keep track of all these observations and within a well defined exchange format and unified procedures, make all the observations available on-line. The links for Eros and are 1998-OR2 are respectively:

Eros observations

and

1998-OR2 observations

It was nice to find a fairly good coverage, and even dense during the flyby of 1998-OR2. Gaia observations, but one, took place between 12h and 17h UTC, or during daylight for European observers, or early morning in America. Despite the extensive ground-based coverage, there are no observations within two or three hours of Gaia's. They all happen the night before, or the following night (European time), and for Eros this is somewhat worse, with even a 2-day lapse before or after Gaia detections. Fortunately Eros is not very close and its motion not too large during this interval meaning that it can be readily interpolated. That would have been more critical with 1998-OR2 at the closest approach.

5.6.1 Complementary 1998 OR2 observations

During the approach to the close encounter of 1998 OR2 with the Earth and Gaia, P. Tanga of the DPAC/CU4 independently secured a small set of observations of 1998-OR2 directly, that were not yet submitted to MPC. Although there was little risk not to have good ground-based astrometry around the critical dates from observatories around the globe (the data flowing regularly to Minor Planet Center) this was seen as an interesting exercice made possible by the

FIGURE 16: Two frames obtained on April 16, 2020, showing the displacement of the asteroid 1998 OR2 between 20:05:50 UT (top image) and 20:32:57 UT (bottom). Exposure time of 2 s, sky North to the left (credit : Paolo Tanga, DPAC/CU4).

availability of a standard Schmidt Cassegrain telescope (35 cm in diameter, f/11), fitted with a CMOS-based camera (model QHY174GPS). This camera has fast acquisition rate capabilities (up to $\sim$100 frames per second) and, in particular, a GPS receiver embedded, offering the interesting capability to accurately time tag the frames (at better than 0.1 ms, depending upon calibrations). Today, this is commonly available, off-the-shelf equipment for the evolved amateurs, of course also well suited to some professional applications (such as the timing of fast

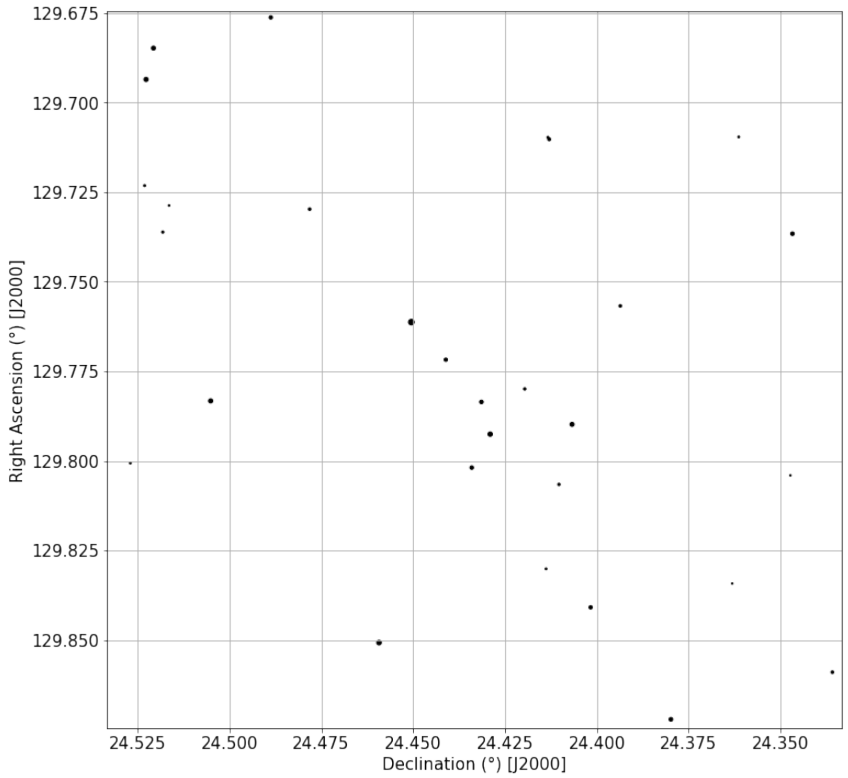

FIGURE 17: Stars extracted from the Gaia DR2 catalogue at the limit G<16 for the central area shown in Fig. 16. Point size represents brightness. The faintest stars present in this extraction are beyond a useful signal-to-noise ratio for the reduction of the astrometry of our short-exposure images (credit : Paolo Tanga, DPAC/CU4).

transients).

The use of a focal reducer resulted in an equivalent focal length F=2260 mm, and a pixel scale of 0.54 as/pixel (square). Over the nights of April 16 and 17, several sequences of images were acquired, without filters. The exposure was fixed at 2 s. Over this time the smearing of the asteroid due to its apparent motion (apparent velocity ∼90 mas/s- remains within the pixel, and candidate reference stars (brighter than the asteroid itself) did not saturate. An example of the obtained images is shown in Fig. 16.

Table 7 shows the astrometry derived from a selection of the best frames over the two nights.

For both nights, a relatively small number of reference stars extracted from Gaia DR2 (Fig. 17) is available (<10) over the field of view (9×15 arcmin^2), given the limited exposure. This results in an internal calibration error for the astrometry of 60 mas on April 16, and 160 mas for April 17 when the minimum number of reference stars was used (4). In such conditions no correction of field distortion was attempted. These 5 observations have been added to the MPC

TABLE 7: Ground-based astrometry obtained by P. Tanga on the nights of April 16 and 17, 2020, for 1998 OR2.

UT	α HH:MM:SS	δ deg
2020-04-16.8818989	08:39:15.669	+24:22:40.18
2020-04-16.8832532	08:39:16.037	+24:22:31.64
2020-04-17.8372944	08:44:25.181	+22:37:49.27
2020-04-17.8391115	08:44:25.734	+22:37:36.45
2020-04-17.8399565	08:44:25.993	+22:37:30.30

set for the passage of the 17 April.

5.6.2 Interpolation over time

Since there are no observations exactly contemporaneous with Gaia, we have to compute these observations from the existing ground-based data. The idea is to bracket Gaia times by observations from the ground, within about one day ($\pm$2h the night before and the next night) for 1998 OR2, and between the 10 and 15 February for the Eros observation of the 13 February 2019. With this selection I found (including the 5 from P. Tanga on 16-17 April 2020),

1. 41 observations of Eros between 2019-02-10.81159 and 2019-02-15.85705
2. 54 observations of 1998-OR2 between 2020-03-08.90271 and 2020-03-10.38575
3. 38 observations of 1998-OR2 between 2020-04-16.88986 and 2020-04-18.11786
4. 51 observations of 1998-OR2 between 2020-04-27.89536 and 2020-04-29.01178

For each observation the station coordinates are available and are required to make the topocentric correction. The numbers above are the observations that have been selected after a comparison to computed coordinates has shown that for some observers, the residuals were anomalous, larger than $1.5''$, while for most of the others this was below $0.5''$. In total about 10 observations were rejected, and within one night, they were in general from a single observing site.

These observations in right ascension and declination are shown in Fig. 18, for the two planets and the four epochs (13 Feb 2019 for Eros, 09 March, 17 and 29 April 2020 for 1998-OR2).

TABLE 8: Interpolated ground-based observations at the same time as Gaia for the ten observations of table 4 (1-2: Eros, 3-10 : 1998-OR2). Observation times are $T_0 + dt$ in julian days TT. This set is computed without topocentric correction, or during the first iteration with the planet assumed to be at very large distance.

obs	T_0 JD	dt day	$\alpha_{\rm P}$ deg	$\delta_{\rm P}$ deg
1	2458528.0	0.020041	86.111891	15.166364
2	2458528.0	0.094054	86.163335	15.116736
3	2458918.0	-0.023336	116.625419	43.197760
4	2458918.0	0.050726	116.612529	43.192722
5	2458918.0	0.226823	116.581585	43.180528
6	2458957.0	-0.136524	130.438496	23.521494
7	2458957.0	0.039557	130.678494	23.196112
8	2458957.0	0.113611	130.781286	23.057260
9	2458968.0	-0.132764	156.754210	-16.705645
10	2458968.0	0.043182	157.450072	-17.687207

The red line is a second degree fit of the trend over the time covered by the selected ground based observations, and the epochs of the Gaia observations are the vertical blue lines. They usually fall in holes, except marginally for the 09 March data, but the fit is remarkably good. The plots shown are done on the topocentric data, but during the iterations of the processing, the fit is recomputed with the topocentric correction. The residuals for the geocentric fit are always below $0.5''$. The fits were computed directly on the right ascension and declination, with a quadratic polynomial, as,

$$x(t) = a + b(t - t_0) + c(t - t_0)^2$$

where t_0 is at noon of the central day, and x is the fitted parameter. Then the position at Gaia time was evaluated with the polynomial.

A gnomonic projection was also used to work with the projected positions on the tangent plane, tangent to the median direction. The interpolation was done on the local plane coordinates instead, and then the computed observation at Gaia time was back projected on the celestial sphere by an inverse gnomonic projection. At the end the results were not significantly different,

TABLE 9: Interpolated ground-based observations at the same times as Gaia for the ten observations of table 4 (1-2: Eros, 3-10 : 1998-OR2). Observation times are $= T_0 + dt$ in julian days TT. This set computed after the topocentric correction, at the last iteration with the converged value of the *au*.

obs	T_0 JD	dt day	$\alpha_{\rm P}$ deg	$\delta_{\rm P}$ deg
1	2458528.0	0.020041	86.110646	15.171114
2	2458528.0	0.094054	86.162081	15.121481
3	2458918.0	-0.023336	116.624963	43.198403
4	2458918.0	0.050726	116.611547	43.193060
5	2458918.0	0.226823	116.580079	43.180125
6	2458957.0	-0.136524	130.464526	23.535686
7	2458957.0	0.039557	130.703081	23.210121
8	2458957.0	0.113611	130.804516	23.071212
9	2458968.0	-0.132764	156.747252	-16.658601
10	2458968.0	0.043182	157.443589	-17.639807

but this is mathematically a better approach, since the coordinates have no physical meaning and could be singular.

Results are summarised in table 10, with fits on the topocentric and geocentric observations, that is to say after the topocentric shift has been removed. This correction uses the distance to the planet and the current best estimate of the astronomical unit to express that distance in the same unit as the position vector of the observer, given in km. Therefore one needs iterations to get the full correction, starting with the assumption that the distance is very large compared to the Earth radius. The overall process converges in 2 or 3 iterations. The first iteration is then equivalent to the simplified processing when one considers that the reported observations are geocentric, or the planet at very large distance.

In table 10 one sees clearly the importance of the topocentric corrections in the quality of the fit, all the more we have a very close approach, when the difference between the topocentric and geocentric directions are the most pronounced. It remains a small anomaly in declination at the closest approach on the 28 April. However changing the fit from a second order polynomial

TABLE 10: Polynomial fit of the ground based observations : RMS of the residuals in arcsec when the fit is done directly on the topocentric coordinates (left) and after the topocentric corrections to bring the observations at the centre of the Earth (right)

		topocentric		geocentric	
	Epoch	ra	dec	ra	dec
1	13 Feb 2019	2.6	1.2	0.3	0.03
2	09 Mar 2020	7.4	5.2	0.2	0.11
3	17 Apr 2020	10.5	8.9	0.3	0.21
4	28 Apr 2020	42.5	12.0	0.5	3.2

to a cubic the RMS of the residuals fall to $0.32''$. The parabolic model is not good enough to represent the combination of the diurnal parallax and the fast motion of 1998-0R2 during the flyby, a fact which can be understood given the declination change of 7° in just a day.

5.7 A worked out example

We illustrate the procedure with a fully detailed example of the first observation of Eros in February 2019.

From table 4, one has the Gaia observation,

$$\begin{aligned} t &= 3330.520041 \\ \alpha_0 &= 84.294157 \\ \delta_0 &= 14.932476 \end{aligned}$$

then the reconstructed observation at the same time without topocentric correction from table 8

$$\begin{aligned} \alpha_1 &= 86.111768 \\ \delta_1 &= 15.166410 \end{aligned}$$

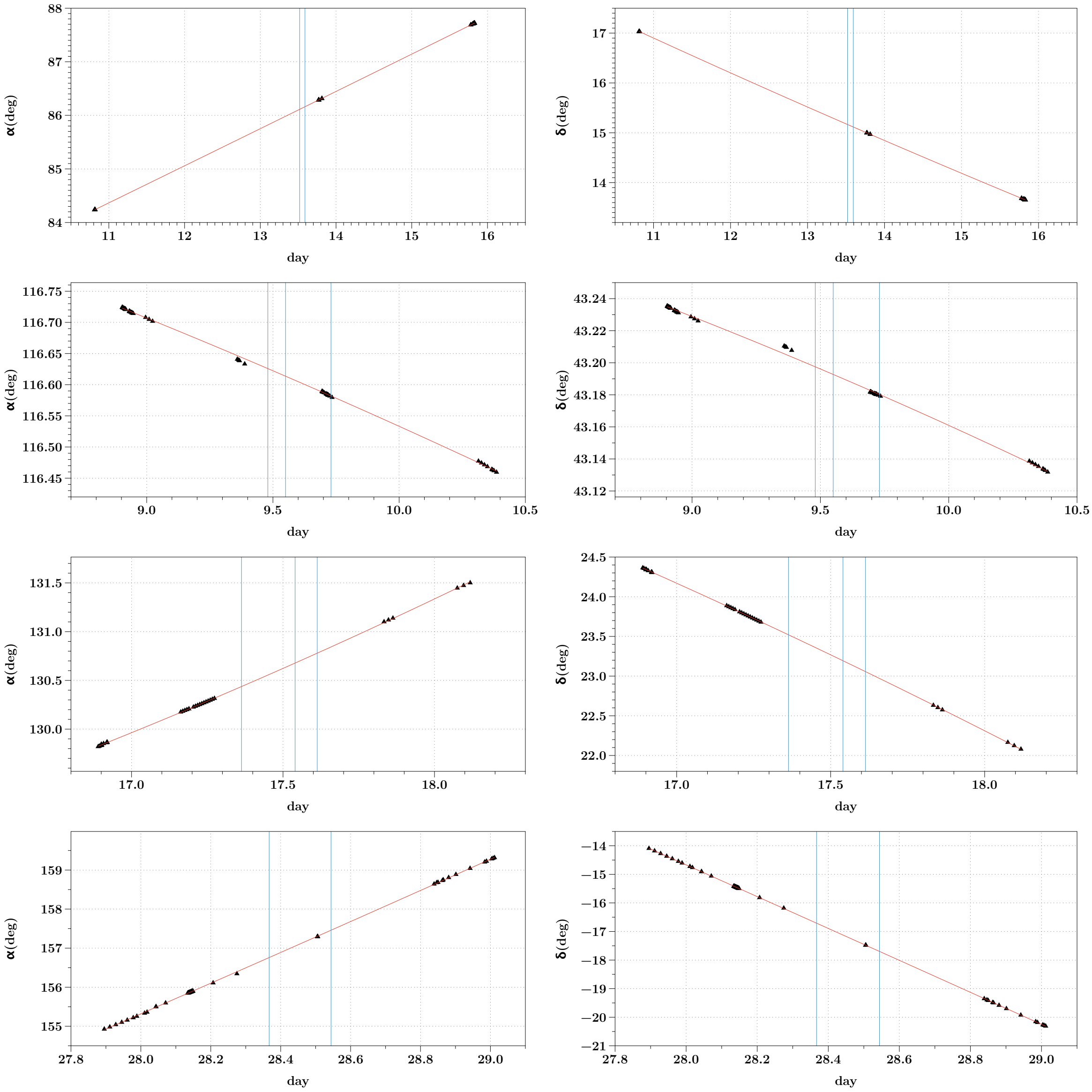

FIGURE 18: Polynomial interpolation of the ground based observations at the Gaia times. From top to bottom : Eros 13 Feb 2019, 1999-OR2 09 March, 17 and 28 April 2020; left : Right ascension, right : declination. Vertical lines mark the epochs of the Gaia observations. Red line : quadratic interpolation. Raw data plotted, before topocentric corrections.

or with the topocentric correction from table 8

$$\alpha_2 = 86.110692$$
$$\delta_2 = 15.171155$$

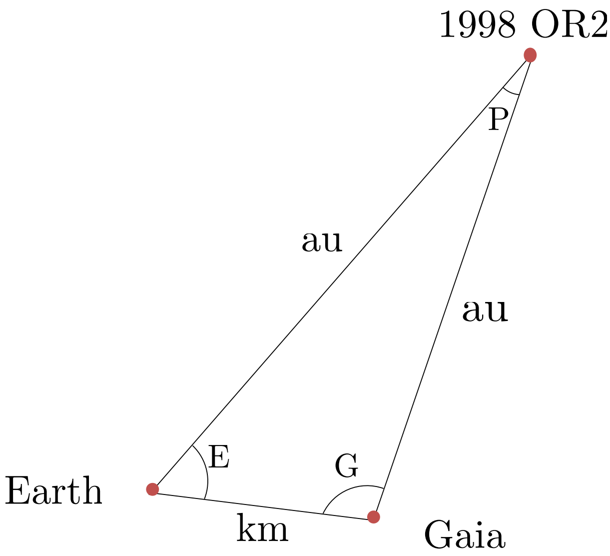

FIGURE 19: The observation triangle showing the measured angles E, G and the known lengths in km or in *au*. From the angular observations and the length of the baseline, one can find the distance EP in km, and then the AU in km.

then the distance of Eros from the Earth centre computed from the orbit of Eros from table 6

$$\rho = 0.24788363\text{au} \tag{10}$$

and finally the position vector of Gaia for this observation taken form table 5,

$$\begin{aligned}
x_{\mathrm{g}} &= -1060167.1 \quad \text{km}\\
y_{\mathrm{g}} &= 799595.6 \quad \text{km}\\
z_{\mathrm{g}} &= 348671.7 \quad \text{km}\\
\rho_{\mathrm{g}} &= 1372909.1 \quad \text{km}
\end{aligned}$$

From these observational and computed data one solves now the triangle of Fig. 19, by computing the angle G and E.

We have the unit vector along GP from α_0, δ_0 as,

$$\begin{aligned}
x_0 &= \cos\alpha_0 \cos\delta_0 = 0.09606379\\
y_0 &= \sin\alpha_0 \cos\delta_0 = 0.96144293\\
z_0 &= \sin\delta_0 = 0.25768051
\end{aligned}$$

and similarly for EP without topocentric correction,

$$\begin{aligned} x_1 = \cos\alpha_1 \cos\delta_1 &= \quad 0.06544854 \\ y_1 = \sin\alpha_1 \cos\delta_1 &= \quad 0.96294844 \\ z_1 = \sin\delta_1 \qquad\quad &= \quad 0.26162339 \end{aligned}$$

and with topocentric shift applied,

$$\begin{aligned} x_2 = \cos\alpha_2 \cos\delta_2 &= \quad 0.06546516 \\ y_2 = \sin\alpha_2 \cos\delta_2 &= \quad 0.96292559 \\ z_2 = \sin\delta_2 \qquad\quad &= \quad 0.26170332 \end{aligned}$$

From scalar products, we have the angles,

$$\begin{aligned} G &= 123.4503914 \\ E_1 &= 54.7788305 \\ P_1 &= 1.7707846 \\ E_2 &= 54.7792300 \\ P_2 &= 1.7703687 \end{aligned} \tag{11}$$

Finally with (3), one computes directly, respectively without (E_1, P_1) or with (E_2, P_2) topocentric correction.

$$\begin{aligned} EP_1 &= 37\,070\,041 \quad \text{km} \\ EP_2 &= 37\,078\,747 \quad \text{km} \end{aligned}$$

and this corresponds to 0.24788363 *au* from (10) and yields the two solutions for the *astronomical unit*,

$$\begin{aligned} \text{AU}_1 &= 149\,546\,142 \quad \text{km} \\ \text{AU}_2 &= 149\,581\,265 \quad \text{km} \end{aligned}$$

instead of

$$au = 149\,597\,870.7 \quad \text{km}$$

or equivalently a relative error $\epsilon = \Delta x / x$ of

$$\begin{aligned} \epsilon_1 &= -3.4 \times 10^{-4} \\ \epsilon_2 &= -1.1 \times 10^{-4} \end{aligned}$$

We could also compute the solar parallax as,

$$\varpi_{\odot_1} = 8.7972''$$
$$\varpi_{\odot_2} = 8.7951''$$

instead of

$$\varpi_{\odot} = 8.7941438''$$

where the last digit in the solutions is not significant. The results are not exactly identical for the last digits with the first line of Table 11 due to the truncated numbers used in this example and a late censoring of a couple of ground based observations, after this step-by-step example was completed.

5.8 Topocentric correction

This is an important issue in this context, since the difference of directions between an observation performed on the surface of the Earth and the direction of the same source, at the same time, from the centre of the Earth can reach nearly $200''$ and is always larger than $20''$. In any case this is much higher than the random error in the ground based observations, which is usually below $0.5''$. However since we fit a set of ground bases observations to compute a synthetic observation at the Gaia time, the contribution of the topocentric shift averages a little between these observations with sometimes positive, sometimes negative shift. But the best remains to do this correction since we know the locations of the observers. Fig. 20 shows the geometry.

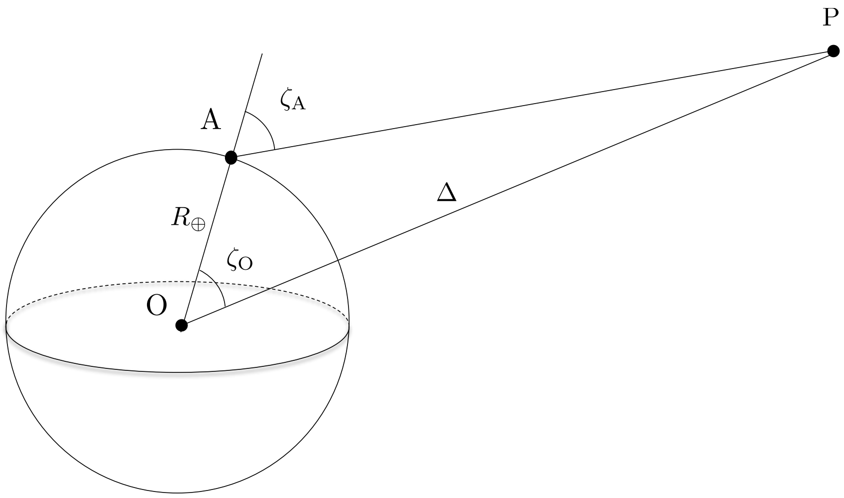

FIGURE 20: Diurnal parallax of a body at finite distance.

Consider first the computation of the topocentric direction from the geocentric direction, that is to say the predictor mode, when one wants to predict a direction from a computed geocentric position. Therefore we know the unit vector $\mathbf{u}_G$ of $\mathbf{OP}$, and also the distance $\Delta = |\mathbf{OP}|$. Now for the topocentric direction,

$$\mathbf{AP} = \mathbf{OP} - \mathbf{OA} \tag{12}$$

and then the unit topocentric vector,

$$\mathbf{u}_\mathrm{T} = \frac{\mathbf{AP}}{|\mathbf{AP}|} \tag{13}$$

With $\mathbf{x}$ for the geocentric position vector of the observer and $R = |\mathbf{x}|$, (very close, but not identical, to $R_\oplus$), one has to first order in the small parameter R/Δ,

$$\mathbf{u}_\mathrm{T} \approx \mathbf{u}_\mathrm{G} - \frac{\mathbf{x}}{\Delta} + \frac{\mathbf{x} \cdot \mathbf{u}_\mathrm{G}}{\Delta}\mathbf{u}_\mathrm{G} \tag{14}$$

The main issue remaining is the fact that the units of $\mathbf{x}$ and Δ are not the same. The former is given in kilometres and the latter in *au* and the scale factor is precisely the goal of the project. Since this is a correction one can start from an approximate value of the *au*, then proceed until a better value is obtained and iterate. The initial value can even be so large as to neglect the correction, and proceed with the first iteration as though the planet were at infinity.

Now to compute the correction in the observed direction, one can either use the vectorial forms (12) or (14) or the trigonometric differential forms as,

$$\begin{aligned}\Delta\alpha\cos\delta &= \frac{X}{\Delta}\sin\alpha - \frac{Y}{\Delta}\cos\alpha \\ \Delta\delta &= \frac{X}{\Delta}\cos\alpha\sin\delta + \frac{Y}{\Delta}\sin\alpha\sin\delta - \frac{Z}{\Delta}\cos\delta\end{aligned}$$

• where $\mathbf{x} = (X, Y, Z)$. This simplifies further with the latitude of the observer and the planet hour angle at observation time,

$$\begin{aligned}\Delta\alpha\cos\delta &= -\frac{R}{\Delta}\sin H\cos\varphi \\ \Delta\delta &= \frac{R}{\Delta}(\cos\varphi\sin\delta\cos H - \sin\varphi\cos\delta)\end{aligned}$$

When R/Δ is not very small, this is the case for the passage of 1998-OR2, the corrector mode is not exactly the predictor mode by just changing the sign. This is true to first order, but this is not sufficient in this context. The distance to the observer is not exactly Δ and must be determined at the same time as the correction is done. So one must resolve a non linear equation with fixed point iteration and the convergence is fast.

5.9 Results

The ten observations were processed as explained in the example, without attempt to consider a global fit of a single parameter, namely the unknown AU, to the set of observations. Basically to stay in line with the demonstration goal of the project, each observation is treated separately and concluded with two estimates of the AU: one with the simple approach and no topocentric correction and the second with the more elaborate model, which includes iterations and the application of a topocentric shift.

TABLE 11: Determination of the mean-distance Earth-Sun in km with the combination of Gaia and ground-based observations, during the close approaches of Eros and 1998-OR2. Results from the simple processing without topocentric correction (left) and with this correction (right). True values of 149 597 870.700 km or $8.794\,143\,836\cdots''$ were used to compute the relative error dx/x.

	not topocentric correction			with topocentric correction		
	AU	$\varpi_\odot$	dx/x	AU	$\varpi_\odot$	dx/x
1	149 537 242	8.79771	-0.41E-03	149 585 467	8.79487	-0.83E-04
2	149 538 791	8.79762	-0.39E-03	149 587 589	8.79475	-0.69E-04
3	149 566 540	8.79599	-0.21E-03	149 598 721	8.79409	0.57E-05
4	149 547 362	8.79711	-0.34E-03	149 598 240	8.79412	0.25E-05
5	149 534 418	8.79788	-0.42E-03	149 596 210	8.79424	-0.11E-04
6	149 976 926	8.77192	0.25E-02	149 598 408	8.79411	0.36E-05
7	149 948 659	8.77357	0.23E-02	149 596 170	8.79424	-0.11E-04
8	149 925 048	8.77495	0.22E-02	149 595 530	8.79428	-0.16E-04
9	149 489 977	8.80049	-0.72E-03	149 602 079	8.79390	0.28E-04
10	149 499 561	8.79993	-0.66E-03	149 602 120	8.79389	0.28E-04

Results are compiled in Table 11 for the ten observations, the first two for Eros and the remainder for the three passages of 1998-OR2. With the full processing, we have a determination of the AU, with the particular meaning given in this note, to within 2×10^{-5} in accuracy. What is expressed is the error by comparison to the true value, and not a statistical precision or probable error, which is not available with this kind of processing. Comparing to the external accuracy of the passage of Eros in 1931, with this simple approach, but a very long baseline, we end up with something about 20 times better. The baseline is 230 times bigger, the ground-based astrometry is probably of comparable accuracy, but the number of observations is much much smaller and the processing limited to an extremely rudimentary modelling.

A Monte Carlo estimate of the random error has been performed, by running the program many times and adding random noise of standard deviation $1''$ to the observations. This gives a statistical scatter much lower than the true errors. Therefore the limitation is not primarily of statistical nature, but in the modelling and the computed quantities. The final conversion of the AU in km, relies on the computed distance to the planet in *au*. To get this value to an accuracy close to one part in a million, requires a perfect numerical integration, excellent orbital parameters and careful time scale management.

At this level of accuracy nothing is trivial. For example, the light-times between the planet and the Earth, and the planet and Gaia are not identical, meaning that the two apparent directions for the same time of arrival of the photons, do not link the observers to the planet at the same geometric positions.

This situation where probable error is too small is on a par with the historical results which were all too optimistic about their external accuracy. This would be similar here had we were looking for a publishable science result. There remain systematic effects that would take too much time to search and this would go beyond the objective of handling the data in a simple manner.

6 Conclusion

We have shown that a sample of Gaia astrometric observations of two planets during their close approach to the Earth, allowed us to evaluate the solar parallax, or the distance to the Sun, as was done until mid XX$^{\text{th}}$ century by professional astronomers. Thanks to the very long baseline between the Earth and Gaia, the parallactic angle reached several degrees during the passages of Eros in 2019 and 1998-OR2 in 2020. Hence the processing of the observations to find out the parallax remains very close to the basic geometric principles, without the complications of a global analysis filled with heavy astronomical computations. Even by neglecting the offset between the centre of the Earth and the true locations of the observing sites, one finds a solar parallax, almost as good as the best determinations done with this technique 70 years ago. A slightly more involved analysis with the application of the topocentric corrections ends up with a solar parallax ~ 20 times better than the best estimate available around 1960.

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