Poisson µStatistics
in High-Energy Astrophysics

Andy Pollock
European Space Agency
ex-*XMM-Newton* RGS Calibration Scientist

ESAC Statistics Workshop 2014 October 27-31

Menú del día

- Poisson v Gaussian
- Calibration
- Rebinning
- Goodness-of-fit
- Point sources
- Spectra
- Background
- Ensembles
- Archives
- M[ai]cro methods
- Mission requirements

Make every photon count.
Understand every photon and every bin.

data ⇔ models

$$\{n_i\}_{i=1,N} \Leftrightarrow \{\mu_i\}_{i=1,N}$$

≥ 0 individual events ⇔ continuously distributed

detector coordinates ⇔ physical parameters

never change ⇔ change limited only by physics

have no errors ⇔ subject to fluctuations

most precious resource ⇔ predictions possible

kept forever in archives ⇔ kept forever in journals and textbooks

Analysis in high-energy astrophysics

data ⇔ models

$$n_i(x,y,t,PI) \Leftrightarrow \mu_i(x,y,t,PI \mid \alpha,\delta,kT,L_X,N_X,....$$

statistics

Statistical nature of scientific truth

- Measurements in high-energy astrophysics collect individual events
- Many different things could have happened to give those events
- Alternatives are governed by the laws of probability
- Direct inversion impossible
- Information derived about the universe is not certain
- Statistical inference quantifies the uncertainties
  - What do we know ?
  - How well do we know it ?
  - Can we avoid mistakes ?
  - What should we do next ?

2 approaches to statistical inference

- Classical or frequentist inference
  - infinite series of identical measurements
  - hypothesis testing and rejection
  - way of the past

- Bayesian inference
  - prior and posterior probabilities
  - way of the present

- Neither relevant for (high-energy) astrophysics
  - one universe
  - irrelevance of prior probabilities and cost analysis
  - choice among many models driven by physics
  - data archives

2 types of statistic

Poisson statistics

Gaussian statistics

2½ types of statistic

- C-statistic ⇔ Poisson statistics
- χ^2-statistic ⇔ Gaussian statistics

Poisson statistics

The Poisson probability distribution for data={*n*≥0} and model={*μ*>0}

$$P(n \mid \mu) = \frac{e^{-\mu}\mu^{n}}{n!}$$

$$\sum_{n=0}^{\infty} P(n \mid \mu) = 1$$

$$\ln P = n \ln \mu - \mu - \ln n!$$

$$\forall n = 0, 1, 2, 3, \ldots, \infty$$

$$P(0 \mid \mu) = e^{-\mu}$$

$$P(1 \mid \mu) = e^{-\mu}\frac{\mu}{1}$$

$$P(2 \mid \mu) = e^{-\mu}\frac{\mu}{1}\frac{\mu}{2}$$

$$P(3 \mid \mu) = e^{-\mu}\frac{\mu}{1}\frac{\mu}{2}\frac{\mu}{3}$$

$$P(n \mid \mu) = P(n-1 \mid \mu)\frac{\mu}{n}$$

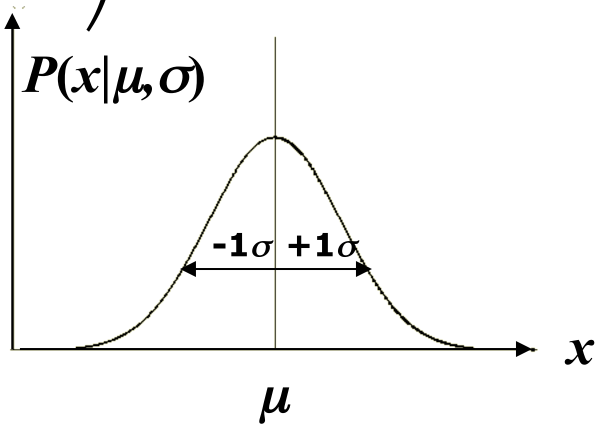
Gaussian statistics

The Normal probability distribution $P(x|\mu,\sigma)$ for data=$\{x \in \Re\}$ and model=$\{\mu,\sigma\}$

$$P(x\,|\,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

1σ 68.3%
2σ 95.45%
3σ 99.730%
4σ 99.99367%
5σ 99.999943%

$$\int_{-\infty}^{+\infty} P(x\,|\,\mu,\sigma)\,dx = 1$$



$$\ln P = -\frac{(x-\mu)^2}{2\sigma^2} - \ln\left(\sigma\sqrt{2\pi}\right)$$

$$\int_{-1\sigma}^{+1\sigma} P(x\,|\,\mu,\sigma)\,dx \approx 0.6827$$

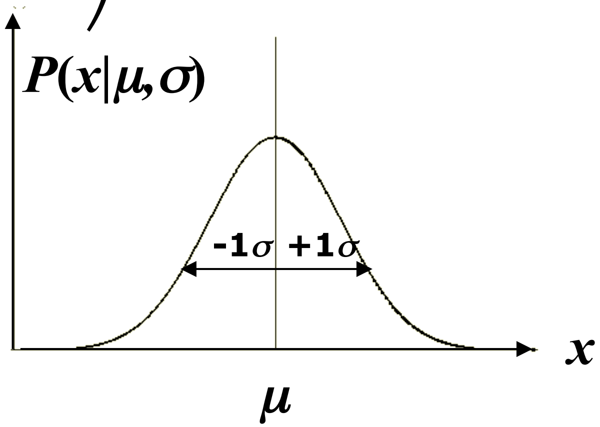
Gaussian statistics

The Normal probability distribution $P(x|\mu,\sigma)$ for data=$\{x \in \Re\}$ and model=$\{\mu,\sigma\}$

$$P(x\,|\,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

1σ 1/3
2σ 1/22
3σ 1/370
4σ 1/15787
5σ 1/1744277

$$\int_{-\infty}^{+\infty} P(x\,|\,\mu,\sigma)\,dx = 1$$



$$\int_{-1\sigma}^{+1\sigma} P(x\,|\,\mu,\sigma)\,dx \approx 0.6827$$

$$\ln P = -\frac{(x-\mu)^2}{2\sigma^2} - \ln\left(\sigma\sqrt{2\pi}\right)$$

Likelihood of data given models

$\{n_i\}_{i=1,N}$ data ⟺ statistics ⟺ models $\{\mu_i\}_{i=1,N}$

$$L = \prod_{i=1}^{N} P(n_i \mid \mu_i)$$

Poisson

$$L = \prod_{i=1}^{N} \frac{e^{-\mu_i}\mu_i^{n_i}}{n_i!}$$

$$\ln L = \sum_{i=1}^{N} n_i \ln \mu_i - \mu_i - \kappa(\ln n_i!)$$

$$-2\ln L = \mathrm{C}$$

Gaussian

$$L = \prod_{i=1}^{N} \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left(-\frac{(n_i-\mu_i)^2}{2\sigma_i^2}\right) dn_i$$

$$\ln L = -\frac{1}{2}\sum_{i=1}^{N}\frac{(n_i-\mu_i)^2}{\sigma_i^2} - \sum_{i=1}^{N}\ln\sigma_i + \kappa(\ln dn_i)$$

$$\sigma_i = \sigma_i(n_i, \mu_i)$$

$$-2\ln L = \chi^2$$

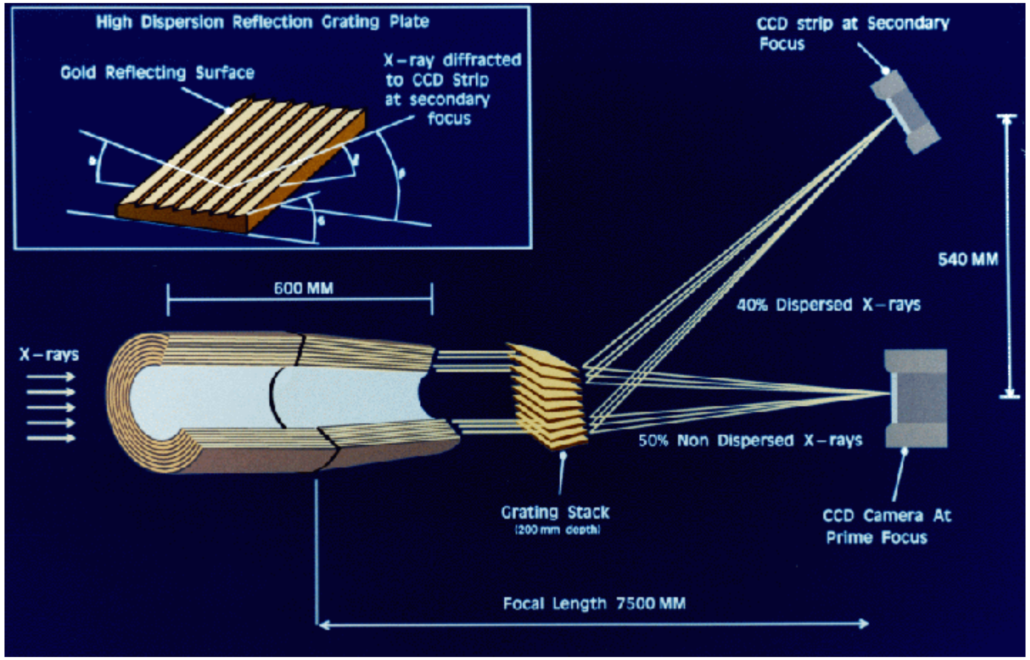
Cash 1979, ApJ, 228, 939

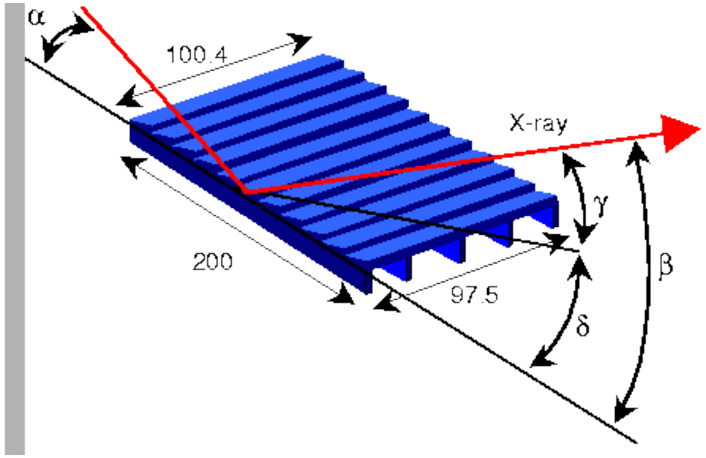
Numerical model of the life of a photon

Detected data are governed by the laws of physics. The numerical model should reproduce as completely as possible every process that gives rise to events in the detector

- photon production in the source (or sources) of interest
- intervening absorption
- effects of the instrument
  - calibration
  - event integrity
    - reconstruction
    - pile-up
- background components
  - cosmic X-ray background
  - local energetic particles
  - instrumental noise
- model it
  - do not subtract it

An *XMM-Newton* RGS instrument

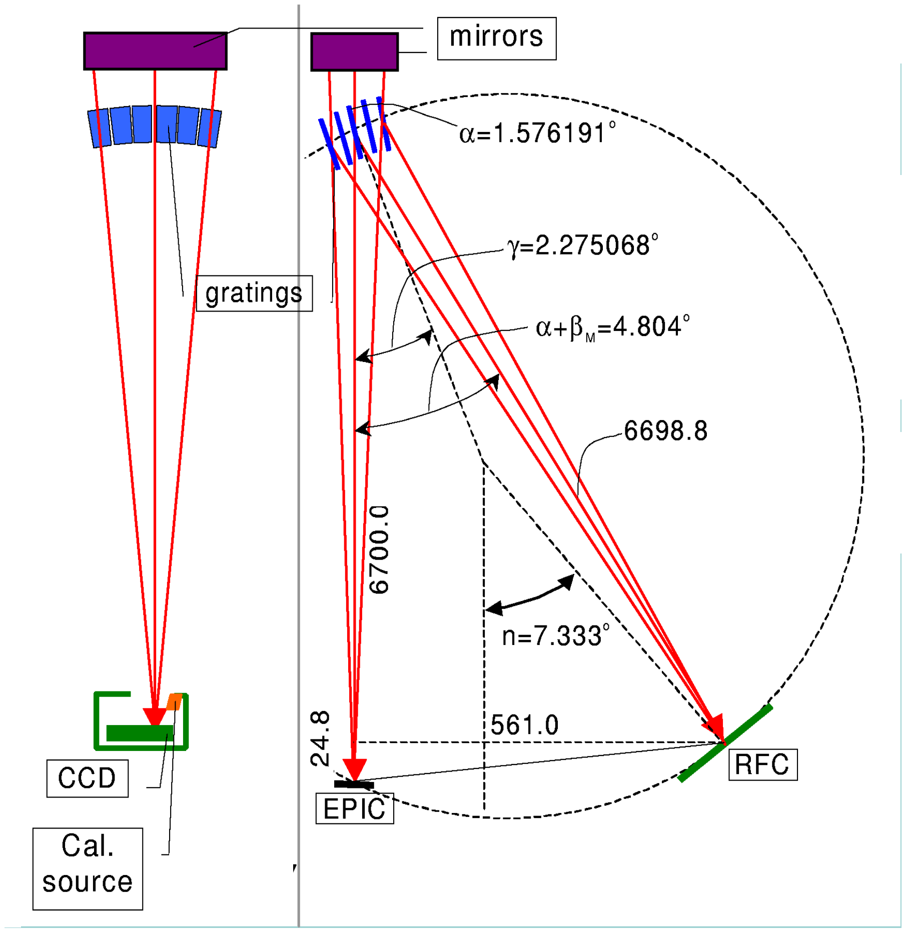




$$\cos\beta = \cos\alpha + m\lambda/d$$

RGS SAS & CCF components

$m\lambda = d(\cos\beta - \cos\alpha)$



BORE.SIGHT
LINCOORDS
MISCDATA

HKPARMINT

ADUCONV
BADPIX
CROSSPSF
CTI

LINESPREADFUNC
QUANTUMEFF
REDIST
EFFAREACORR

rgsproc

- atthkgen
- rgsoffsetcalc
- rgssources
- rgsframes
- rgsbadpix
- rgsevents
- evlistcomb
- gtimerge
- rgsangles
- rgsfilter
- rgsregions
- rgsspectrum
- rgsrmfgen
- rgsfluxer

5-10% accuracy is a common calibration goal

The final data model

$$\mu(\underline{\theta},\underline{\beta},\underline{\Delta},\underline{D})=S(\underline{\theta}(\underline{\Omega}))\otimes R(\underline{\Omega}<\underline{\Delta}>\underline{D})+B(\underline{\beta}(\underline{D}))$$

$\underline{D}$ = set of detector coordinates {X,Y,t,PI,…}
S = source of interest
$\underline{\theta}$ = **set of source parameters**
R = instrumental response
$\underline{\Omega}$ = set of physical coordinates $\{\alpha,\delta,\tau,\upsilon,\ldots\}$
$\underline{\Delta}$ = set of instrumental calibration parameters
B = background
$\underline{\beta}$ = set of background parameters

$\Rightarrow \ln L(\underline{\theta},\underline{\beta},\underline{\Delta}) \Rightarrow \ln L(\underline{\theta}|\underline{\beta},\underline{\Delta}) \Rightarrow \ln L(\underline{\theta})$

Calibration obligations to $\ln L(\underline{\theta}|\underline{\beta},\underline{\Delta})$

- point-spread functions *aka* PSF
- line-spread functions *aka* LSF
- sensitivity
- background
- stability

Bias in data analysis

Maximum-likelihood estimates, μ, of the mean counts for observations {n}

- χ^2 data weights $\quad \Sigma(n-\mu)^2/n \quad \rightarrow \quad \mu^{-1} = \langle n^{-1}\rangle$
- C-statistic $\quad \Sigma n/n\mu-\mu \quad \rightarrow \quad \mu = \langle n\rangle$ (the correct answer)
- χ^2 model weights $\quad \Sigma(n-\mu)^2/\mu \quad \rightarrow \quad \mu^2 = \langle n^2\rangle$

Biases for Poisson distribution with μ = 100

- $1/\langle n^{-1}\rangle = 98.9897$
- $\langle n\rangle = 100$
- $\sqrt{\langle n^2\rangle} = 100.4988$

- Bias is binning dependent
- !*cf* Sivia, Data Analysis: a Bayesian tutorial

Bias in data analysis

Maximum-likelihood estimates, μ, of the mean counts for observations {n}

- χ^2 data weights $\Sigma(n-\mu)^2/n$ → $\mu^{-1} = \langle n^{-1}\rangle$
- C-statistic $\Sigma n\,ln\mu-\mu$ → $\mu = \langle n\rangle$ (the correct answer)
- χ^2 model weights+ $\Sigma(n-\mu)^2/\mu+ln\mu$ → $\mu = \langle n\rangle$ (the correct answer)

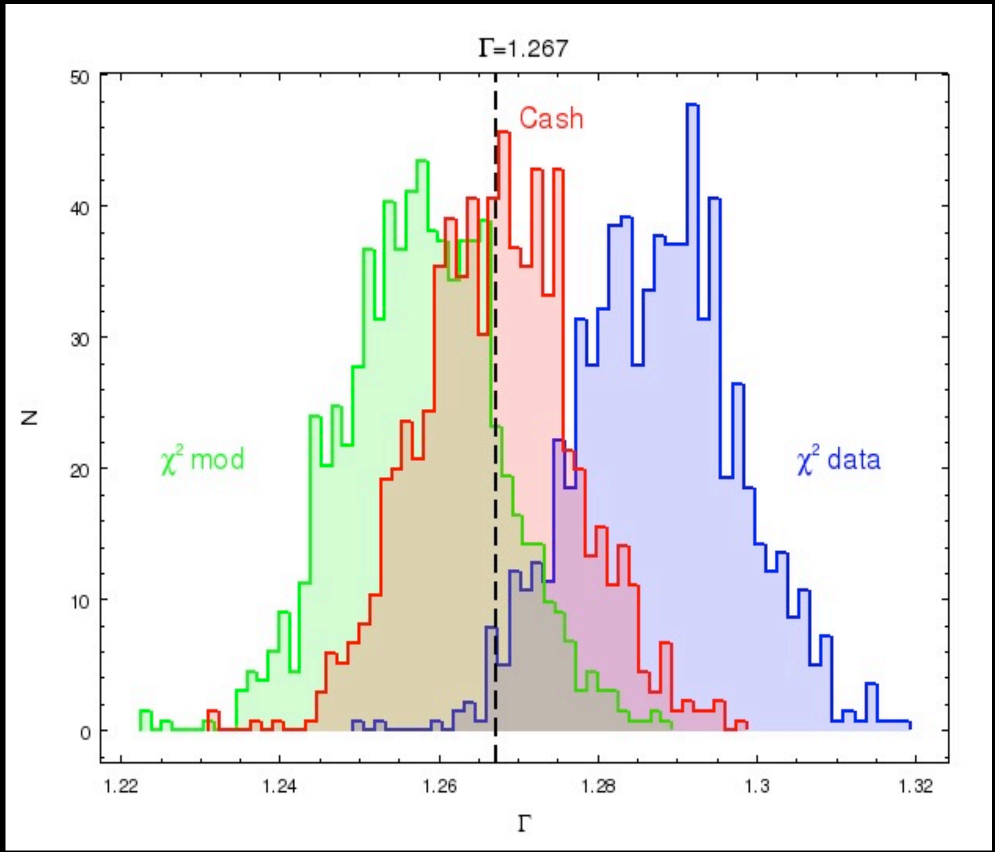
Biases for Poisson distribution with μ = 100

- $1/\langle n^{-1}\rangle = 98.9897$
- $\langle n\rangle = 100$
- $\sqrt{\langle n^2\rangle} = 100.4988$

- Bias is binning dependent
- Unbias is binning independent

Kashyap's 2011 IACHEC talk

scary plot #1: bias



If the likelihood is not appropriate, you may not get the *best* fit.

[A. Siemiginowska]

RGS-pn rectification alternatives

XSPEC statistic	RGS1	RGS2	RGS1	RGS2
χ^2(data)	-2.8%	-2.7%	+0.1%	+0.2%
C	-0.4%	-0.2%	+3.9%	+3.3%
χ^2(model)	+1.2%	+1.5%	+5.0%	+5.6%
λ	short	short	long	long

Choice of statistical method makes a difference.

First commandment of data analysis

① Use Poisson statistics to explore parameter space

Poisson likelihood

$$\ln L = \sum_{events} \ln \mu - \sum_{bins} \mu$$

Poisson maximum-likelihood condition

data $\{n_i\}$ photons : model $\{\mu_i = sp_i + b\}$: XSF p_i : unknown $\{s, b\}$

$$\ln L = \sum_{i=1}^{N} n_i \ln \mu_i - \mu_i$$

$$= \sum_{i=1}^{N} n_i \ln\left(sp_i + b\right) - (sp_i + b)$$

$$\frac{\partial \ln L}{\partial s} = \sum_{i=1}^{N} \frac{n_i p_i}{sp_i + b} - p_i = 0$$

$$\frac{\partial \ln L}{\partial b} = \sum_{i=1}^{N} \frac{n_i}{sp_i + b} - 1 = 0$$

$$s \frac{\partial \ln L}{\partial s} + b \frac{\partial \ln L}{\partial b} = \sum_{i=1}^{N} \frac{n_i sp_i}{sp_i + b} - sp_i + \sum_{i=1}^{N} \frac{n_i b}{sp_i + b} - b = 0$$

$$\sum_{i=1}^{N} n_i = s \sum_{i=1}^{N} p_i + b \sum_{i=1}^{N} 1$$

Other Poisson properties

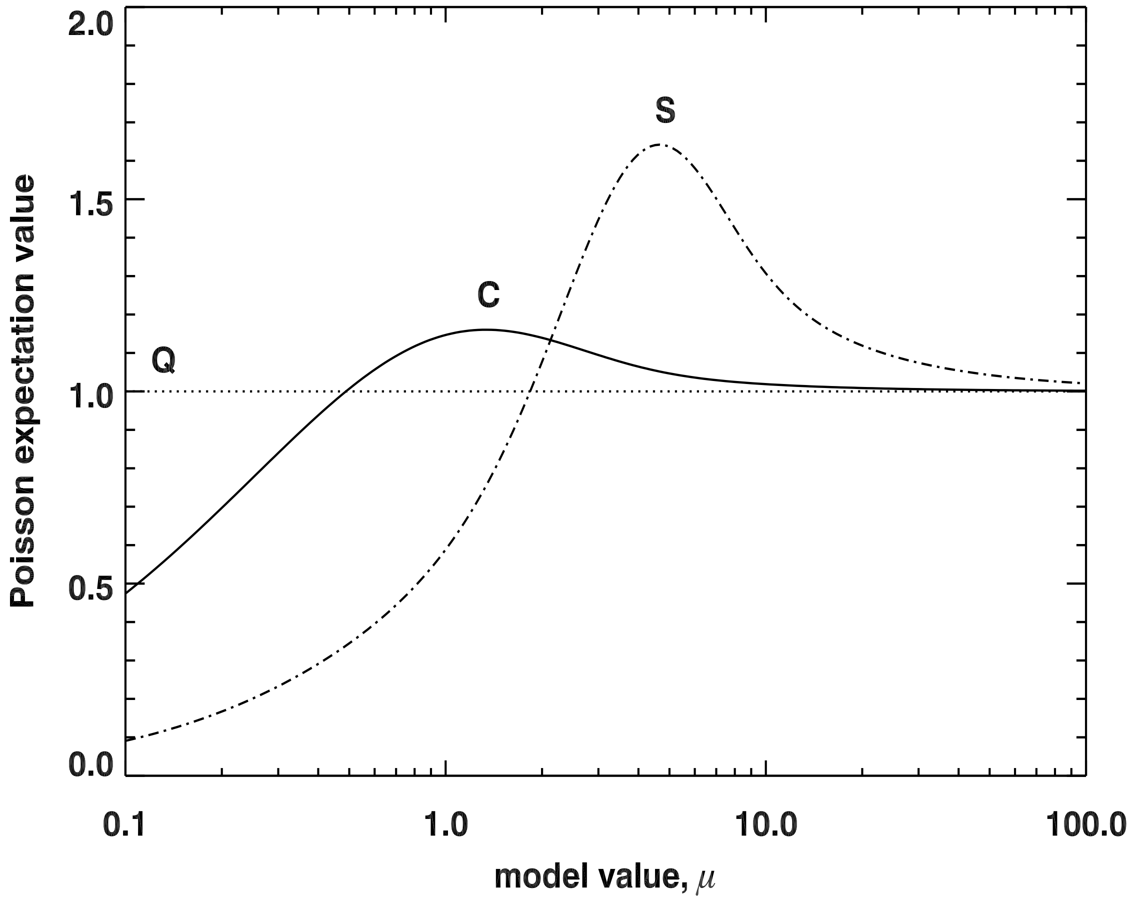
- Sum of Poisson variables is a Poisson variable
    - Binning irrelevant
- Inhomogenous Poisson process
    - rate(t)
        - *e.g.* Restaurant customers

First commandment of data analysis

① Use Poisson statistics to explore parameter space

Goodness-of-fit is a separate issue

χ^2 data $S_i = (n_i - \mu_i)^2/n_i$: C-statistic $C_i = n_i \ln \mu_i - \mu_i$: χ^2 model $Q_i = (n_i - \mu_i)^2/\mu_i$



$$\langle Q \rangle = \langle (n-\mu)^2/\mu \rangle = \langle n^2 \rangle/\mu - 2\langle n \rangle + \mu\langle 1 \rangle = 1$$

First commandments of data analysis

① Use Poisson statistics to explore parameter space

② Use Pearson’s statistic for goodness-of-fit

Uses of the log-likelihood $\ln L(\underline{\theta})$

- $\ln L$ is what you need to assess all and any data models
  - Frequentists and Bayesians agree
  - Locate the global maximum-likelihood model when $\underline{\theta}=\underline{\theta}^*$
    - should be independent of MinMax method
    - beware local traps
  - Compute the goodness-of-fit statistic, Q
    - $Q/\nu \sim 1$ ideally
      - ν = number of degrees of freedom
  - Estimate model parameters and uncertainties with $\ln L(\underline{\theta})$
    - $\underline{\theta}^* = \{p_1, p_2, p_3, p_4, \ldots, p_M\}$
    - calculate 1σ intervals
    - investigate parameter dependences
  - Investigate the whole multi-dimensional surface $\ln L(\underline{\theta})$
    - make lots of plots
  - Inspect data and model
    - pay attention to every bin

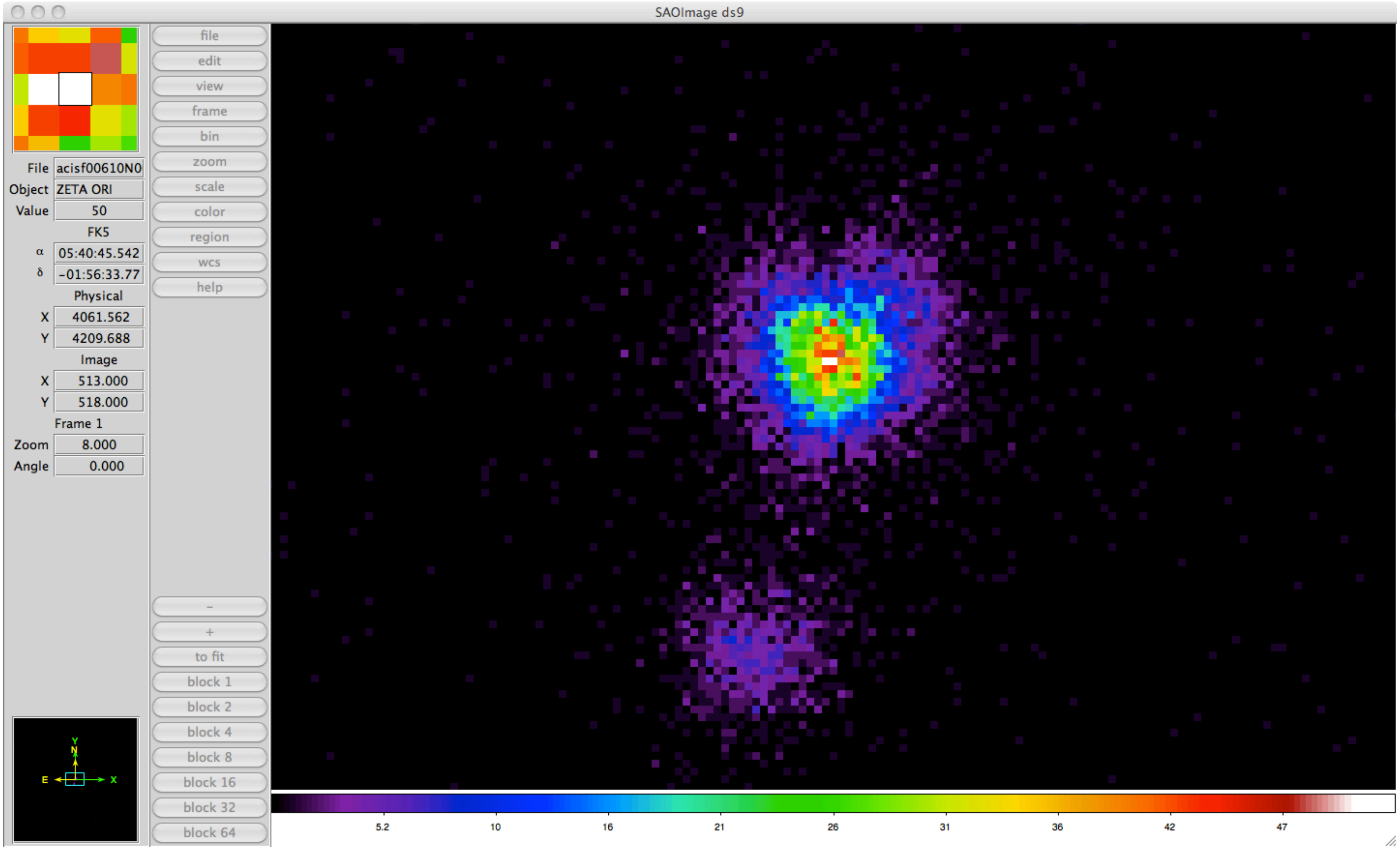
Calibrating $\Delta \ln L(\underline{\theta})$

- $2\Delta \ln L \Leftrightarrow \sigma\sqrt{2\Delta \ln L}$
  - $2\Delta \ln L < 1.$ is not interesting
  - $2\Delta \ln L > 10.$ is worth thinking about (*e.g.* 2XMM DET_ML ≥ 8.)
  - $2\Delta \ln L > 100.$ Hmmm...

Practical considerations

- Q/ν is rarely ~ 1
- Solution often dominated by systematic errors
  - no-one knows the right way (except perhaps PyBlocks)
- Formal probabilities are not taken too seriously
  - $Q/\nu > 2$ is bad
  - $Q/\nu \sim 1$ is good
  - $Q/\nu \sim 0$ is also bad

Poisson analysis of point-sources μ(x,y)



Point-source inference

IDL journal — tcsh — 103×18

idl | IDL journal — tcsh | IDL code — ...acs-21.4.15 | IDL console — tcsh | pgxwin_server

```
showCXOTargetHistory,target='ADS4263',cluster=['A','B'],episodeList=zOCXOeL
;                          MJD-START  MJD-STOP    x       y     T(s)    Count Rate(/s)      lnL
;------------+-----------+---------+---------+------+------+------+---------------+---------+
;CHANDRA ACIS    610       51642.195 51642.894 4061.5 4209.6  59670  0.1560(0.0017)   41963.4 ADS4263 A
;                                              4059.7 4205.0  59670  0.0138(0.0005)    1622.3 ADS4263 B
;CHANDRA ACIS    1524      51643.873 51644.035 4060.3 4209.0  13785  0.1396(0.0032)    9876.9 ADS4263 A
;                                              4058.1 4204.1  13785  0.0130(0.0010)     403.4 ADS4263 B
;CHANDRA ACIS    13460     55894.174 55895.850 3999.1 4153.3 142896  0.1184(0.0009)  101747.9 ADS4263 A
;                                              3996.7 4146.7 142896  0.0044(0.0002)     443.7 ADS4263 B
;CHANDRA ACIS    13461     55905.885 55906.506 3985.3 4113.3  53000  0.1199(0.0015)   37092.6 ADS4263 A
;                                              3982.8 4106.9  53000  0.0044(0.0004)     215.3 ADS4263 B
;CHANDRA ACIS    14373     55905.007 55905.552 3985.2 4113.2  46425  0.1133(0.0016)   31413.1 ADS4263 A
;                                              3982.8 4106.9  46425  0.0053(0.0004)     235.0 ADS4263 B
;CHANDRA ACIS    14374     55901.200 55901.379 3985.2 4113.4  15337  0.1107(0.0028)    9992.4 ADS4263 A
;                                              3982.8 4106.9  15337  0.0057(0.0007)      94.3 ADS4263 B
;CHANDRA ACIS    14375     55908.441 55908.864 3985.4 4112.7  36062  0.1216(0.0019)   24586.6 ADS4263 A
;                                              3982.9 4106.9  36062  0.0047(0.0004)     163.9 ADS4263 B
;------------+-----------+---------+---------+------+------+------+---------------+---------+
```

Point-source calibration obligations

```
Terminal — idl — 97×12
idl | IDL journal — more | IDL code — ...acs-21.4.15 | IDL console — tcsh | pgxwin_server

IDL> showCXOTargetHistory,target='WR134'
                                 MJD-START  MJD-STOP    x        y     T(s)   Count Rate(/s)      lnL
------------+-----------+---------+---------+------+------+------+---------------+---------+
CHANDRA ACIS    8909     54506.444 54506.677 4115.2 4065.6  19301  0.0433(0.0016)    3462.9 WR134
------------+-----------+---------+---------+------+------+------+---------------+---------+
IDL> psf=xllPSF(path='/data/Chandra/CSC/8909/acisf08909_000N001_r0002b_psf3.fits.gz')
IDL> showCXOTargetHistory,target='WR134',psf=psf
                                 MJD-START  MJD-STOP    x        y     T(s)   Count Rate(/s)      lnL
------------+-----------+---------+---------+------+------+------+---------------+---------+
CHANDRA ACIS    8909     54506.444 54506.677 4115.2 4065.6  19301  0.0409(0.0015)    4611.4 WR134
------------+-----------+---------+---------+------+------+------+---------------+---------+
IDL> 
```

Point-source goodness-of-fit

```
IDL> xllShowSourceFit,episode=WR134CSCeL[0]
-------+-------+-------+-------+-------+-------+-------+
   0   |   0   |   5   |   3   |   1   |   2   |   0   |
   0.6|   0.2|   0.5|   1.0|   0.9|   1.2|   0.2|
-------+-------+-------+-------+-------+-------+-------+
   0   |   3   |  13   |  40   |  19   |   1   |   1   |
   0.7|   1.2|   3.4|  11.9|   8.8|   2.9|   0.8|
-------+-------+-------+-------+-------+-------+-------+
   1   |   1   |  39   | 159   | 125   |  19   |   1   |
   0.5|   2.5|  23.6| 156.7|  77.9|  10.7|   0.7|
-------+-------+-------+-------+-------+-------+-------+
   0   |   4   |  36   | 117   |  94   |  22   |   2   |
   0.9|   4.3|  32.5| 217.4|  98.3|  13.8|   2.1|
-------+-------+-------+-------+-------+-------+-------+
   1   |   0   |   7   |  23   |  17   |   5   |   0   |
   0.7|   1.6|   6.2|  29.2|  20.5|   3.4|   1.0|
-------+-------+-------+-------+-------+-------+-------+
   0   |   1   |   2   |   4   |   4   |   1   |   0   |
   0.1|   0.7|   1.9|   3.0|   1.2|   0.7|   0.5|
-------+-------+-------+-------+-------+-------+-------+
   0   |   1   |   0   |   0   |   1   |   1   |   1   |
   0.4|   0.3|   0.6|   0.6|   0.6|   0.6|   0.2|
-------+-------+-------+-------+-------+-------+-------+
Q(49)=271.8
```

Point-source historical obligations

1987ApJ...320

Einstein IPC X-RAY OBSERVATIONS OF WOLF-RAYET STARS

WR	Name	IPC field	Date	t_{obs} (100s)	λ	h_-	h_*	h_+	L_X (0.2–4.keV) (10^{32}ergs s^{-1})
						counts per 100s			
5	HD17638	5041	1979.47	61	0.		0.	0.1	0. ± 4.
6	HD50896	2281	1979.79	31	176.1	4.5	5.1	5.6	9.0 ± 0.9
		7872	1980.22	101	334.8	3.2	3.5	3.8	6.2 ± 0.5
		2282	1981.30	42	54.9	1.4	1.7	2.1	3.0 ± 0.6
11	γ Vel	2284	1979.84	32	137.7	5.1	6.0	6.8	1.1 ± 0.2
12	MR13	736	1980.43	20	0.		0.	0.5	0. ± 29.
16	HD86161	5077	1979.97	31	5.5	0.1	0.4	0.8	11. ± 7.
17	HD88500	10058	1981.07	55	0.		0.	0.1	0. ± 8.
18	HD89358	3012	1979.96	22	0.		0.	0.4	0. ± 5.
21	HD90657	3342	1979.53	19	1.7	0.	0.5	1.4	5.8 ± 9.6
22	HD92740	3139	1979.53	22	12.7	0.7	1.3	2.0	9.0 ± 3.
		4222	1979.53	49	14.5	0.6	1.1	1.5	8.1 ± 4.
		776	1978.98	118	3.1	0.1	0.3	0.6	2.7 ± 2.
24	HD93131	3141	1979.53	17	3.9	0.2	0.9	1.8	8.1 ± 3.7
		4223	1979.53	41	10.5	0.6	1.1	1.8	
25	HD93162	3141	1979.53	17	129.9	12.	14.	15.	137. ± 9.
		4222	1979.53	49	257.0	11.	12.	14.	
		4223	1979.53	41	193.4	12.	13.	15.	
28	MS2	1167	1978.98	14	0.		0.	0.5	0. ± 35.
30	HD94305	10059	1981.07	37	0.9	0.	0.2	0.5	15. ± 29.
38	MS8	2161	1979.53	113	1.2	0.	0.2	0.5	1.7 ± 2.7
40	HD96548	2285	1979.53	21	0.		0.	0.3	0.7 ± 3.4
		3009	1979.53	9	0.	0.	0.	0.4	
		7873	1980.64	61	0.2	0.	0.0	0.2	
46	HD104994	5042	1980.11	56	11.0	0.2	0.4	0.7	35. ± 18.
47	HDE311884	7256	1980.65	21	3.3	0.1	0.6	1.1	19. ± 15.
48	θ Mus	5956	1980.65	36	103.0	3.0	3.5	4.0	20. ± 3.
54	MR48	7257	1980.65	17	0.8	0.	0.3	0.8	27. ± 50.
57	HD119078	5044	1980.07	65	0.		0.	0.1	0. ± 9.
67	MR55	775	1979.68	23	0.5	0.	0.6	1.3	8. ± 10.
		7925	1980.62	59	0.9	0.	0.2	0.6	
78	HD151932	3140	1979.16	21	5.3	0.1	0.5	0.8	2.6 ± 1.7
79	HD152270	5075	1980.25	18	2.1	0.0	0.9	1.9	4.4 ± 4.4
97	HDE320102	2552	1979.73	21	26.3	1.4	2.0	2.6	34. ± 10.
98	HDE318016	2552	1979.73	21	0.		0.	0.4	0. ± 4.
101	DA3	2550	1979.73	21	0.		0.	0.3	
102	LSS4368	5045	1980.25	48	2.1	0.0	0.2	0.5	13. ± 13.
104	MR80	2170	1979.24	17	0.7	0.	0.4	1.3	3.8 ± 8.2
105	AS268	4671	1979.69	17	3.7	0.1	0.8	1.5	8.2 ± 6.6
111	HD165763	5959	1981.23	43	1.0	0.	0.2	0.5	0.4 ± 0.8
113	CV Ser	5960	1981.20	79	4.6	0.0	0.2	0.5	2.0 ± 1.5
122	NaSt1	3490	1979.23	34	0.7	0.	0.2	0.6	35. ± 70.
124	M1–67	7417	1981.27	34	0.1	0.	0.1	0.3	2. ± 12.
125	MR93	8680	1981.27	57	39.1	0.9	1.3	1.6	14. ± 4.
134	HD191765	5046	1979.90	99	23.5	0.5	0.7	1.0	4.6 ± 1.6
		3137	1979.90	9	3.3	0.1	0.6	1.1	
135	HD192103	5046	1979.90	99	0.3	0.	0.1	0.2	0.3 ± 1.1
		3137	1979.90	9	0.		0.	0.6	
136	HD192163	827	1979.27	111	2.0	0.	0.1	0.3	0.6 ± 0.6
137	HD192641	5963	1980.33	45	5.0	0.1	0.4	0.7	1.6 ± 1.1
138	HD193077	3495	1979.27	53	23.6	0.6	1.0	1.4	4.6 ± 1.6
139	V444 Cygni	7875	1980.27	109	116.4	1.2	1.5	1.7	7.7 ± 1.3
144	MR110	Σ_5	1978.96	183	0.		0.	0.1	0. ± 1.7
145	AS422	3378	1978.96	54	10.3	0.5	0.9	1.4	8.4 ± 3.2
		3389	1978.96	54	3.8	0.1	0.6	1.0	
		3381	1978.96	30	1.8	0.	0.4	1.0	
		3388	1978.96	24	0.9	0.	0.3	0.9	
		3387	1978.96	25	1.1	0.	0.4	1.1	
146	MR112	3384	1978.96	51	6.3	0.2	0.6	1.1	9.4 ± 5.9
147	AS431	5995	1979.92	52	127.0	2.6	3.0	3.4	47. ± 6.
148	HD197406	7874	1980.39	99	0.		0.	0.2	0. ± 14.
152	HD211654	4558	1979.95	14	2.4	0.0	0.4	0.8	9.1 ± 8.9
154	HD213049	10061	1981.07	43	0.3	0.	0.1	0.3	1.1 ± 4.0
155	CQ Cep	1319	1980.53	19	6.7	0.3	0.7	1.1	14. ± 9.
for comparison from EXOSAT (Pollock 1987)									
140	HD193793		1984.45						400. ± 40.

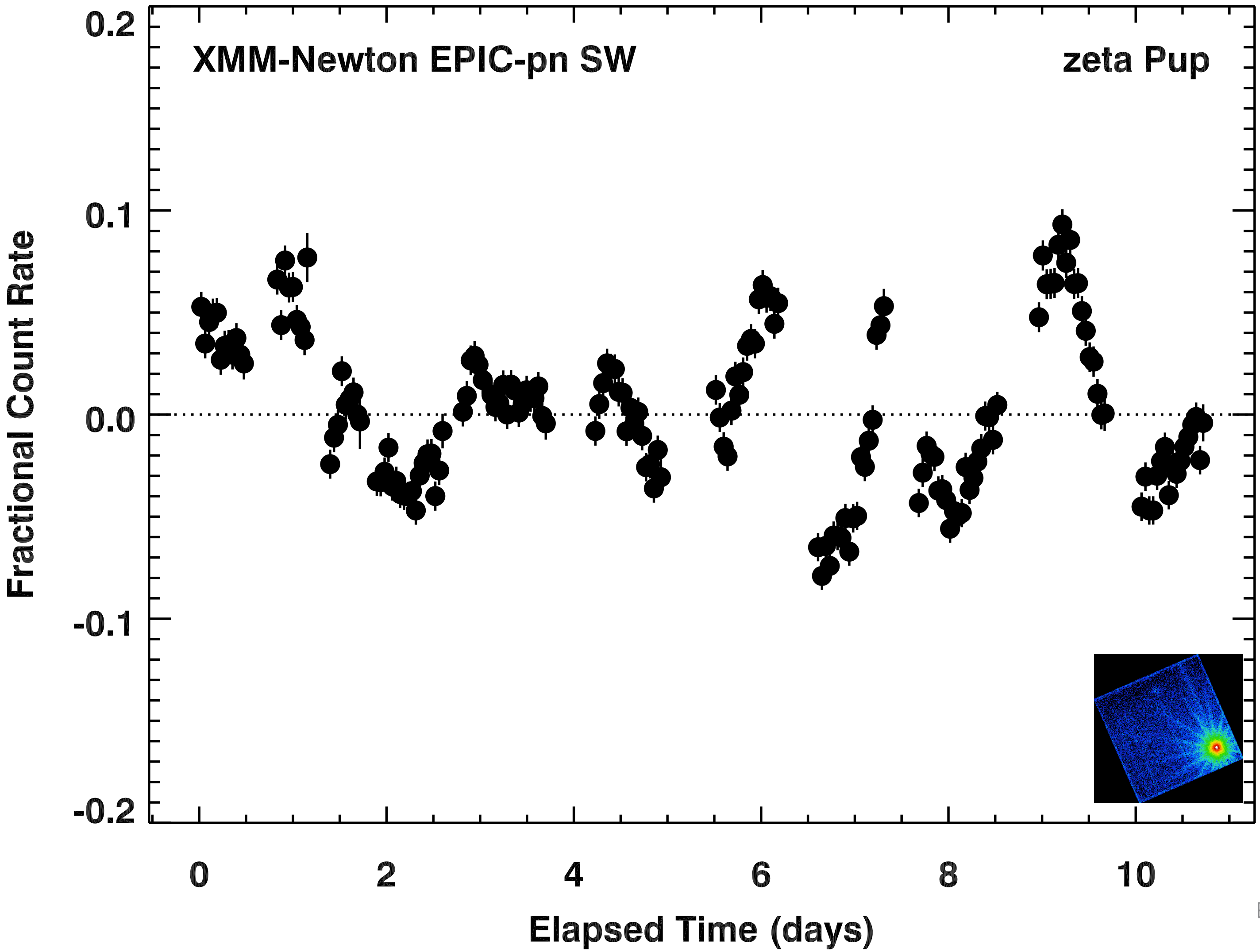
Point-source historical obligations

Einstein IPC X-Ray Observations of Wolf-Rayet Stars

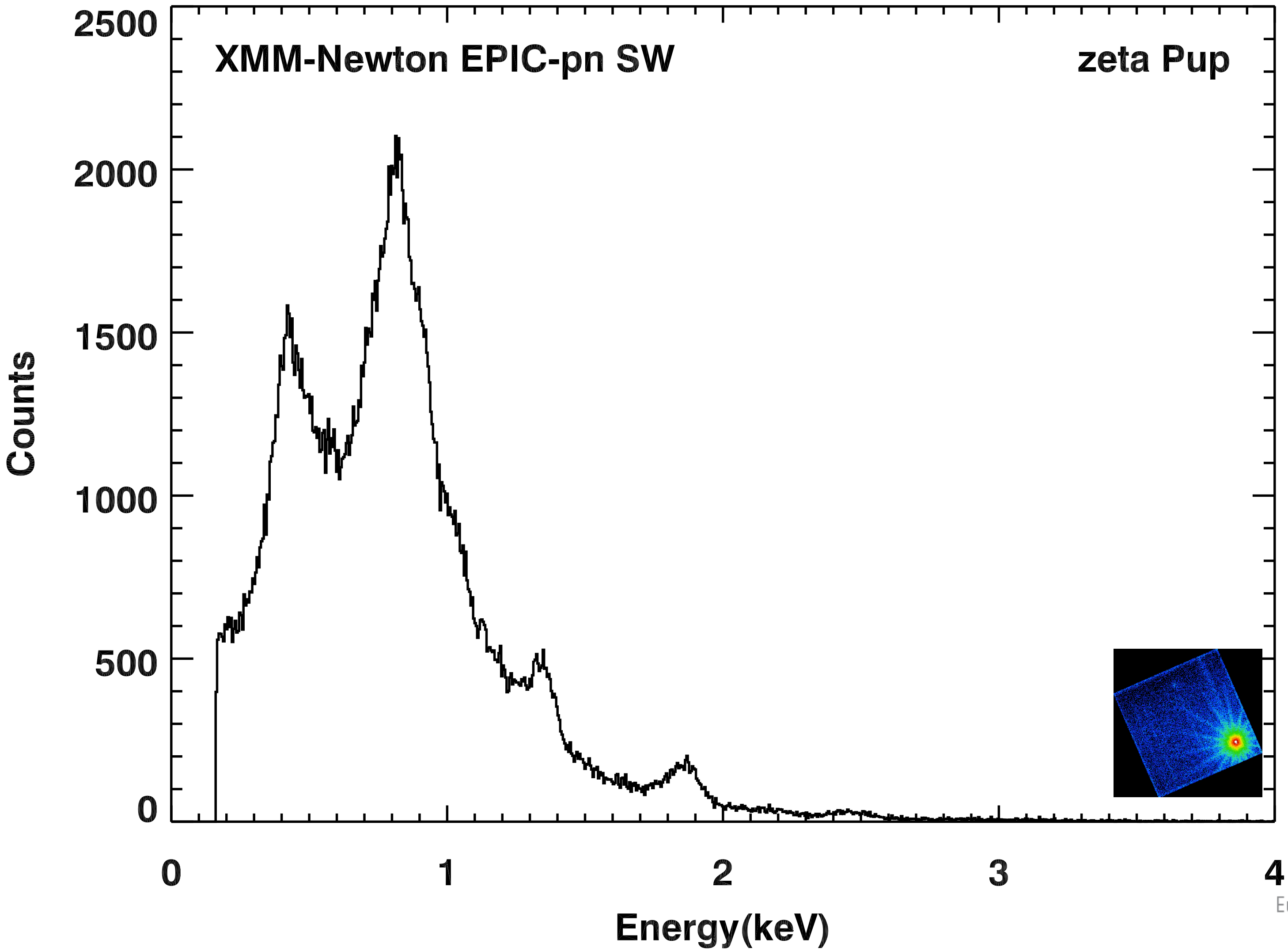
WR	Name	IPC field	Date	t_{obs} (100s)	λ	h_-	h_*	h_+	L_X (0.2–4.keV) (10^{32}ergs s^{-1})
						counts per 100s			
134	HD191765	5046	1979.90	99	23.5	0.5	0.7	1.0	4.6 ± 1.6
		3137	1979.90	9	3.3	0.1	0.6	1.1	

```
IDL> showCXOTargetHistory,target='WR134',psf=psf
                         MJD-START  MJD-STOP     x       y     T(s)    Count Rate(/s)     lnL
------------+-----------+---------+---------+------+------+------+---------------+---------+
CHANDRA ACIS    8909     54506.444 54506.677 4115.2 4065.6  19301  0.0409(0.0015)    4611.4 WR134
------------+-----------+---------+---------+------+------+------+---------------+---------+
```

Point-source variability μ(x,y,t)

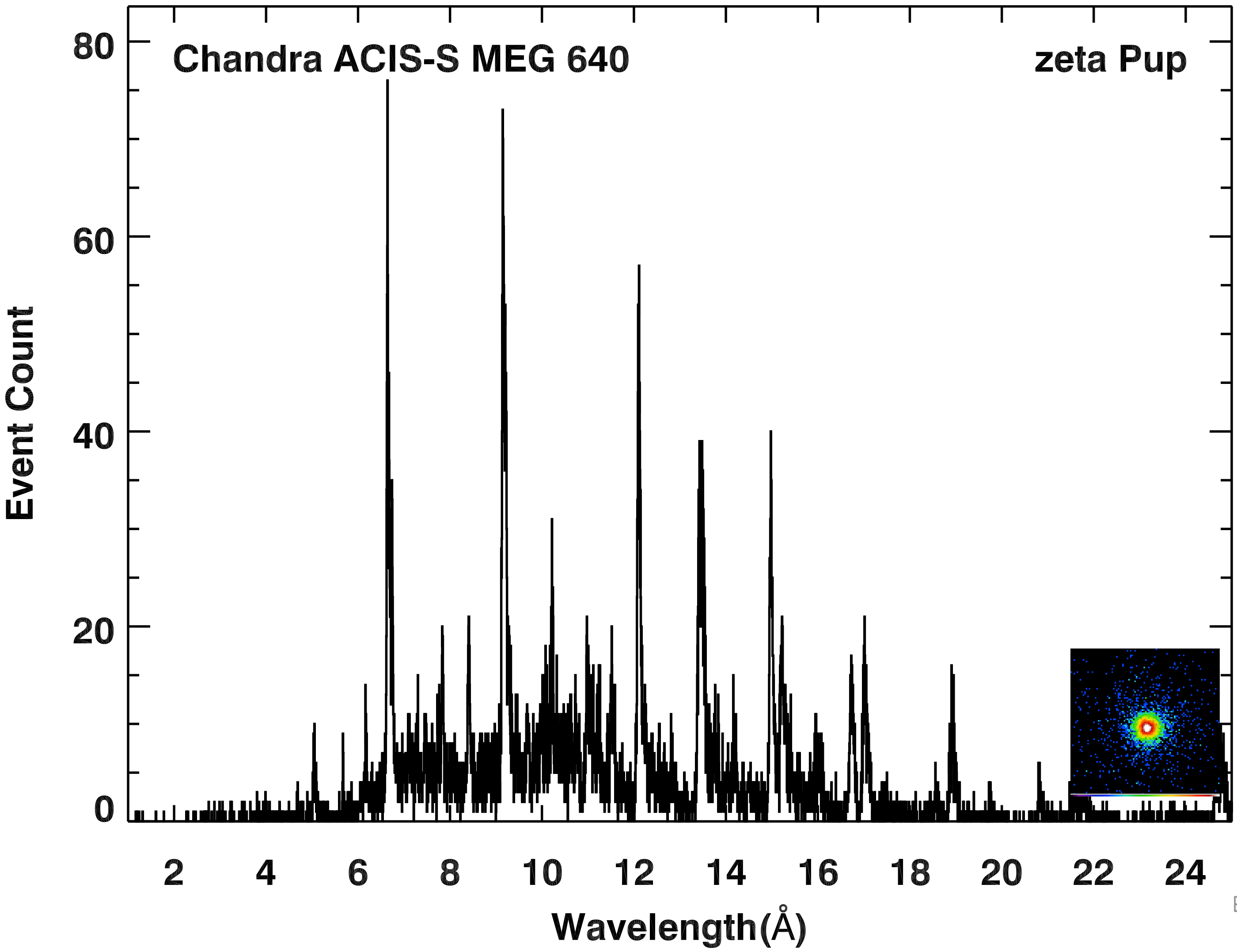


Point-source spectrum μ(E)

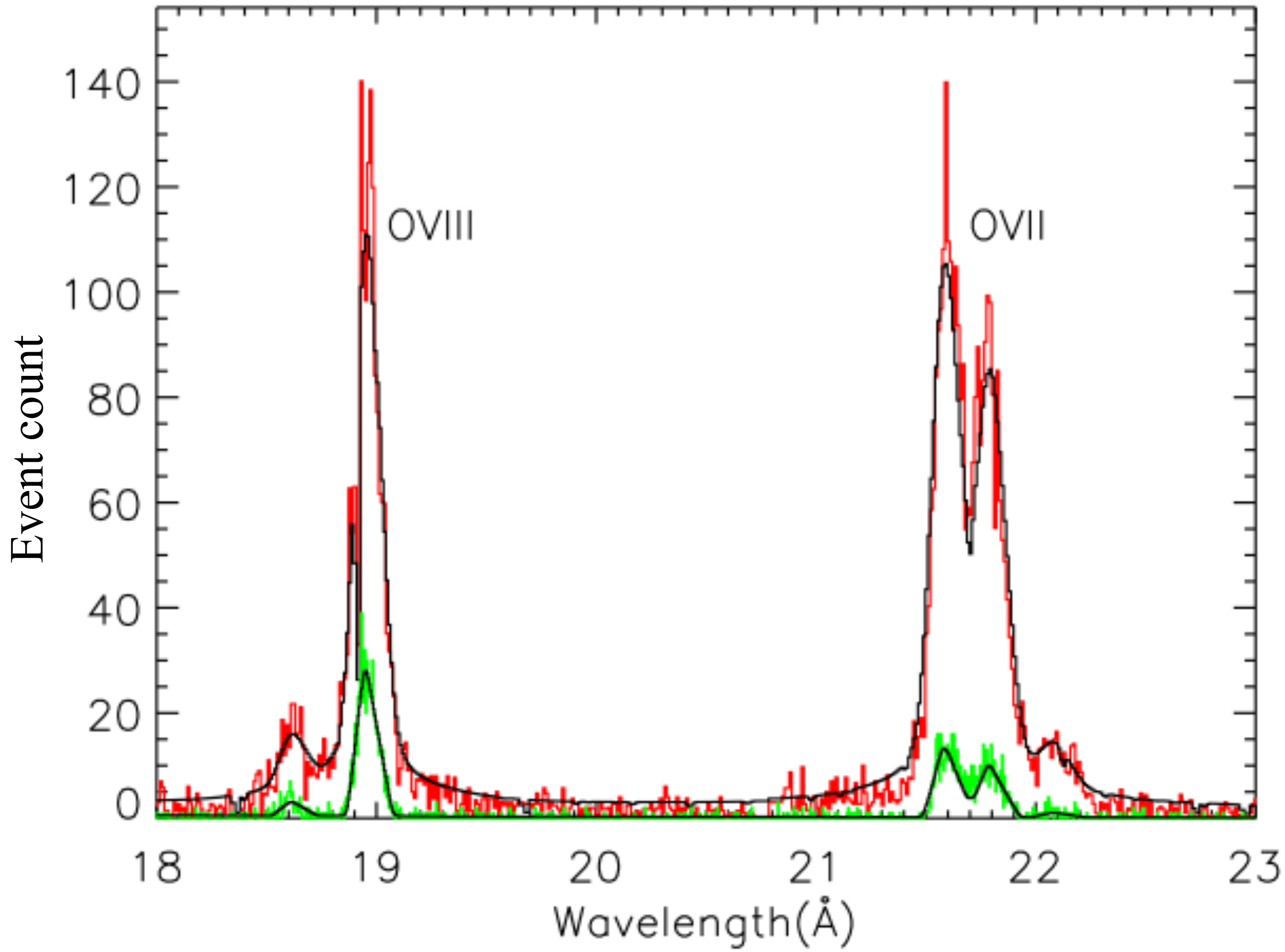


Point-source spectrum μ(λ)

esa



Poisson analysis of point-source spectra



ζ Orionis

XMM RGS

Chandra MEG

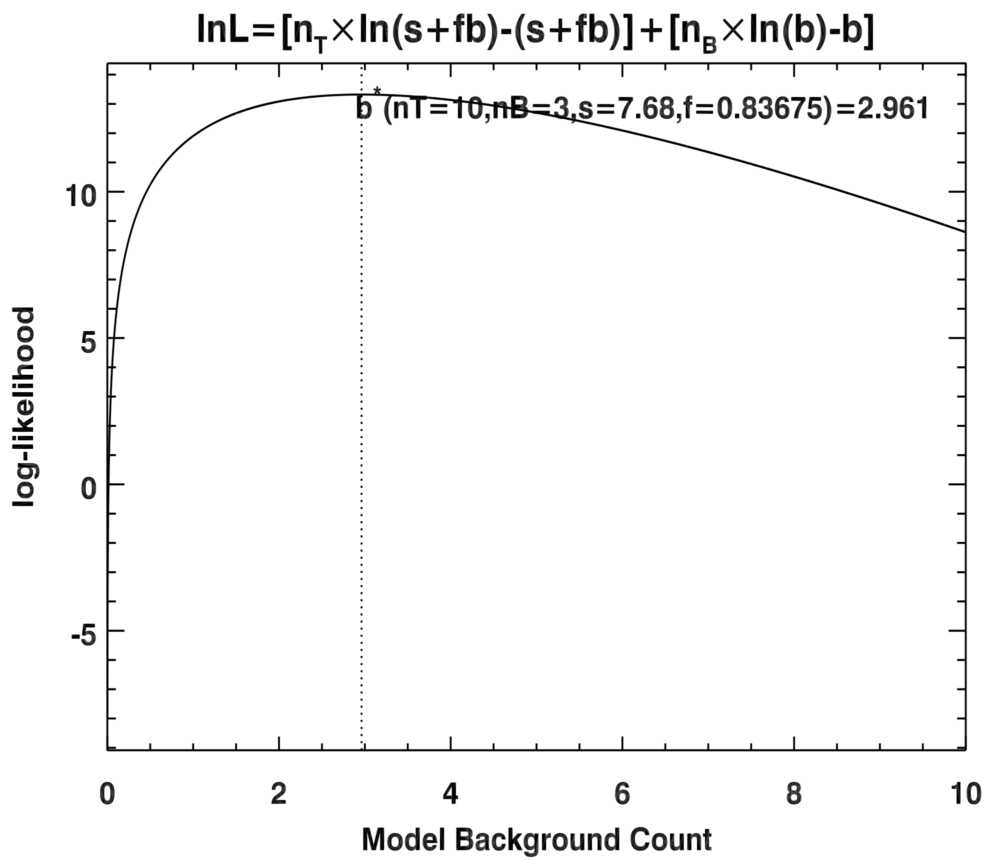
To rebin or not to rebin a spectrum ?

- Pros
  - Gaussian ≡ Poisson for $n \gg 0$
  - dangers of oversampling
  - saves time
  - everybody does it
  - "improves the statistics"
  - `grppha` and other tools exist
  - on log-log plots $\ln 0 = -\infty$
- Cons
  - rebinning throws away information
  - 0 is a perfectly good measurement (*cf* 4'33")
  - images are never rebinned
  - Poisson statistics robust for $n \geq 0$
  - $\mu_1 + \mu_2$ is also a Poisson variable
  - oversampling harmless
  - adding bins does not "improve the statistics"

Leave spectra alone! Don't rebin. Use Poisson statistics.

General-purpose background method

```
nT ⇒ mT = mS + fB×mB
nB ⇒ mB
      fB×(nT/mT-1) + (nB/mB-1) = 0
      cf Wachter et al. 1979, ApJ, 230, 274
```

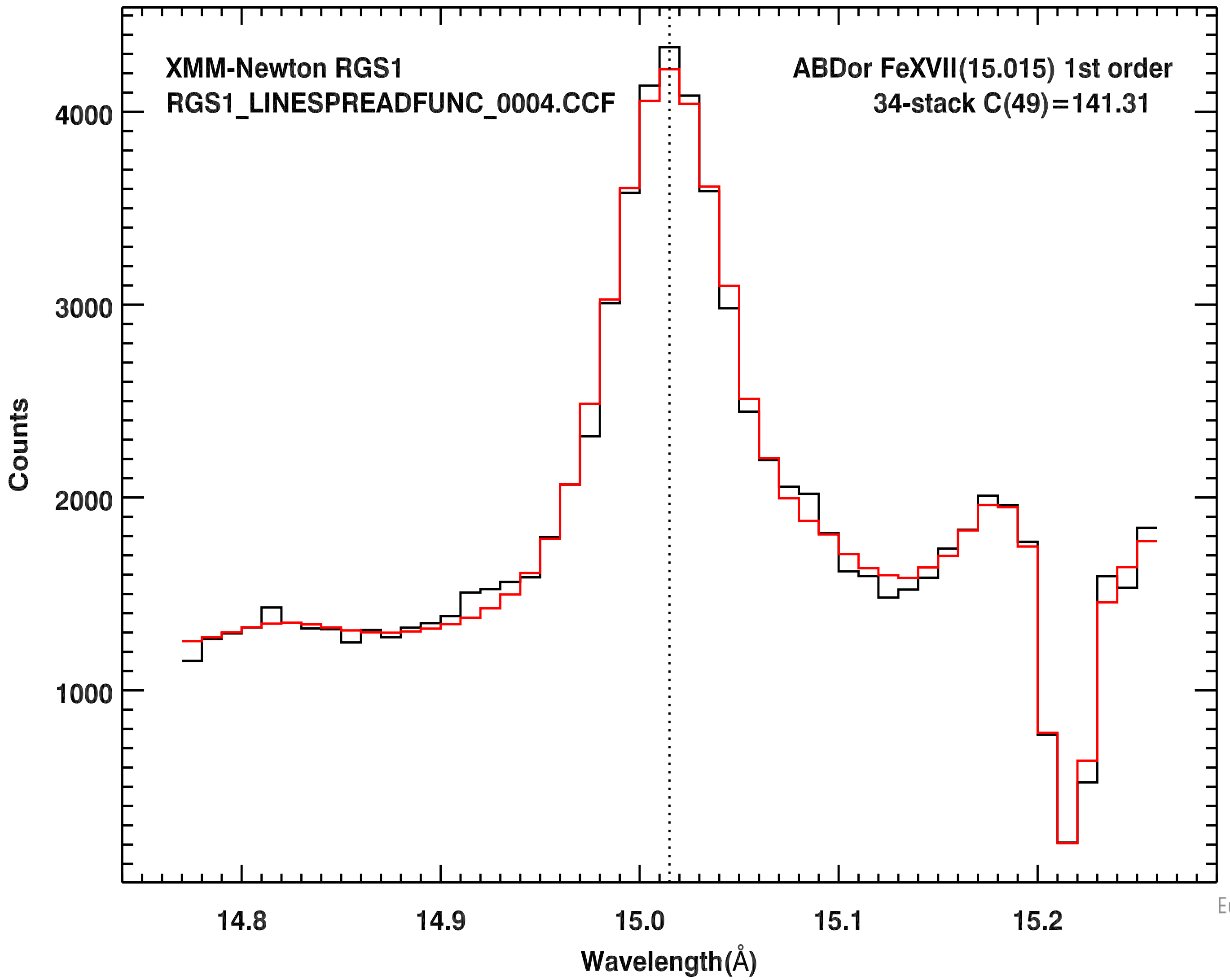


Model stacks

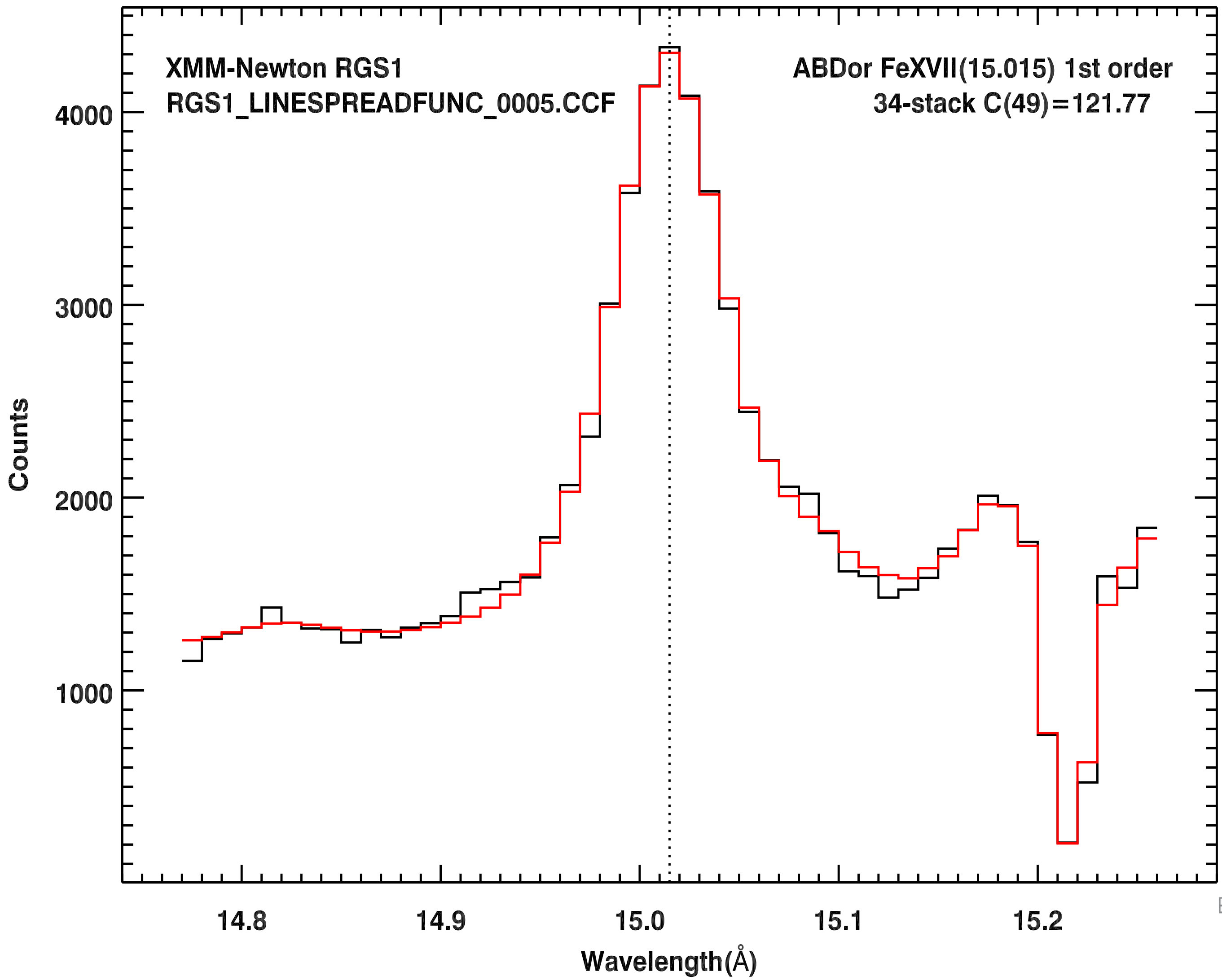
```
XSPEC> fit
XSPEC> exportXspecModelDetails XMD.fits

    CHANNEL
    E_MIN (keV)
    E_MAX (keV)
    QUALITY
    XSPECHAN
    EXPOSURE (s)
    AREA (cm²)
    NOTICED
    DATA aka nT
    BKGDATA aka nB
    BKGRATIO aka fB
    MODEL aka mT
    SRCMODEL aka mS
    BKGMODEL aka mB
    DCSTAT = 2[nT×ln(nT/mT)-(nT-mT)] + 2[nB×ln(nB/mB)-(nB-mB)]
```

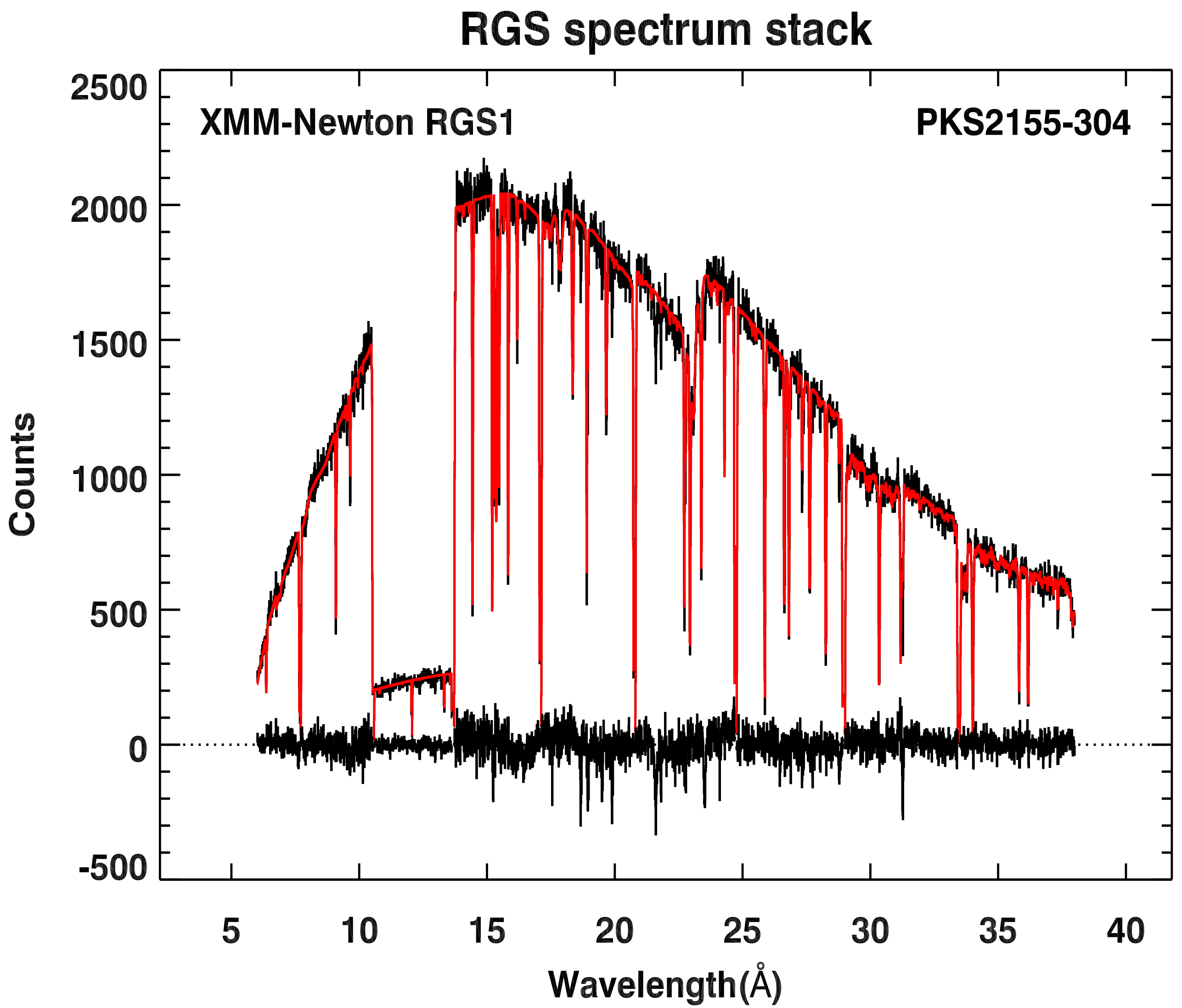
RGS1 AB Dor FeXVII line with an old LSF



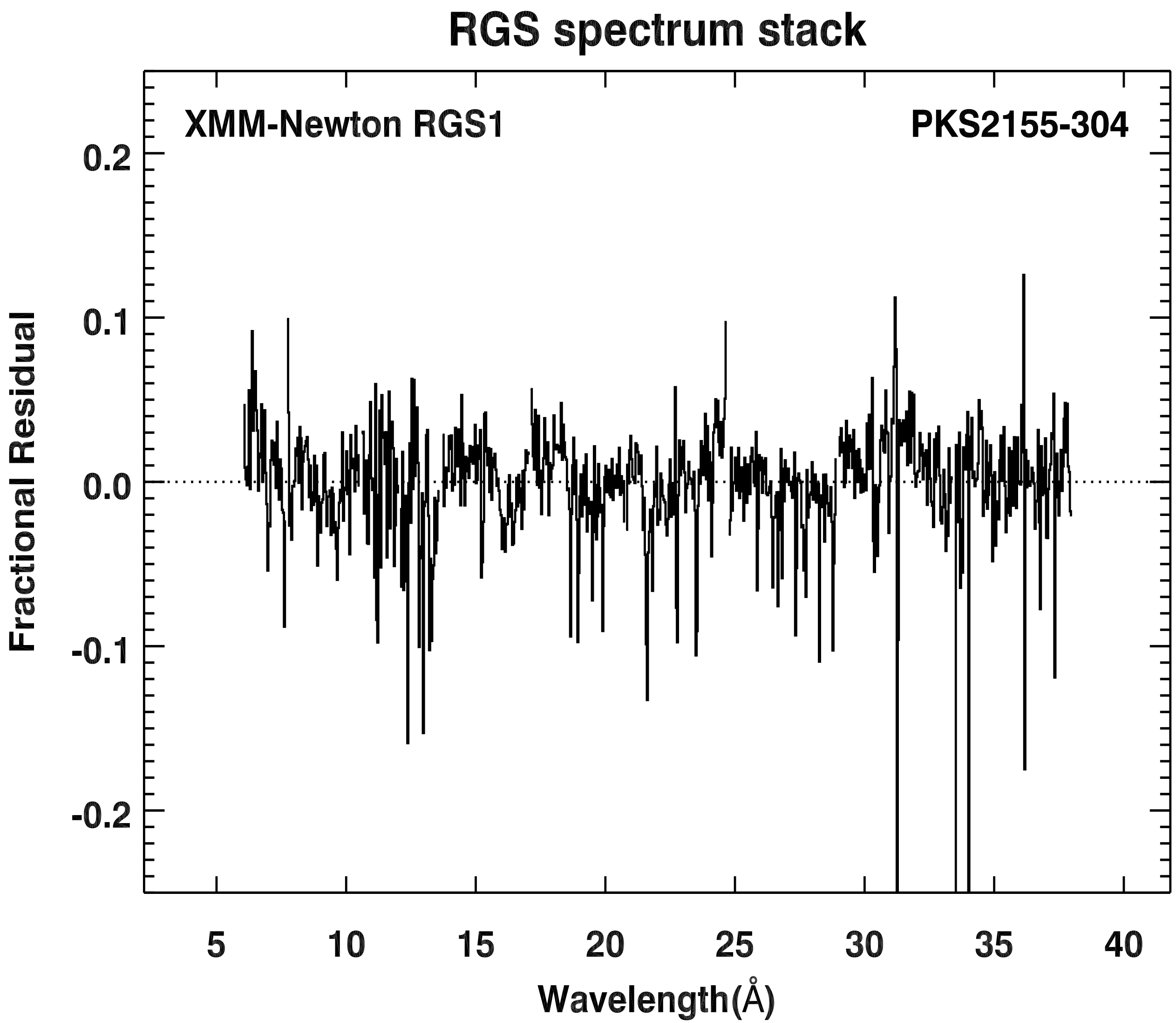
RGS1 AB Dor FeXVII line with a new LSF



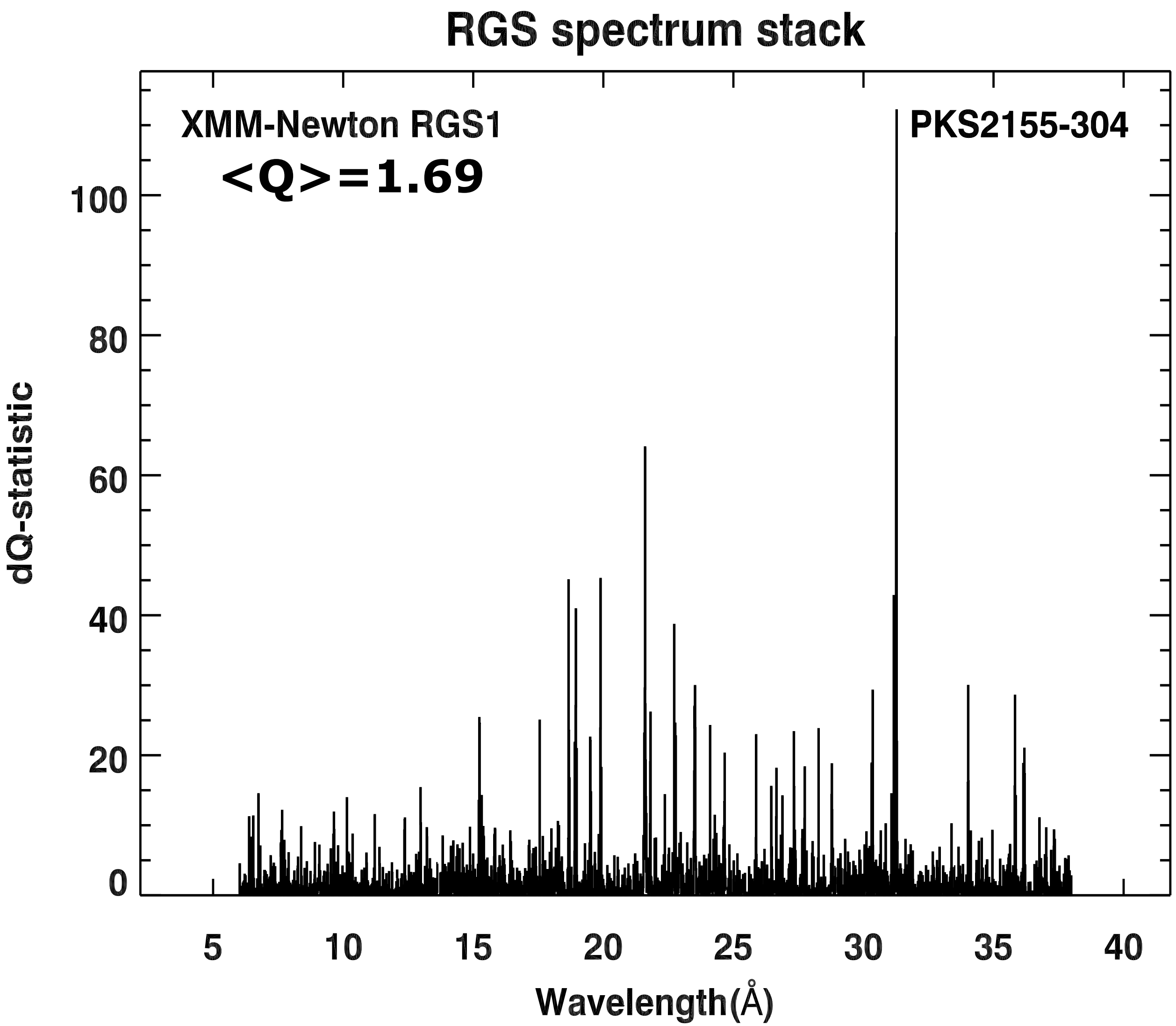
RGS data statistics



RGS systematic errors



RGS statistics : $Q=(n-\mu)^2/\mu$



RGS statistics of individual channels

PKS2155-304 1349_0411780201 RGS1 1st order

chan	nT	mT	mS	nB	mB	f	resid	dC	qT	qB
1041	111	132.92	126.30	9	7.91	0.83693	-20.83	3.97	3.61	0.15
1042	105	104.65	102.20	3	3.01	0.81564	+0.34	0.00	0.00	0.00
1044	0	1.81	1.81	0	0.00	0.93844	-1.81	3.62	1.81	0.00
1045	55	42.11	37.17	4	5.51	0.89720	+11.37	4.05	3.94	0.42
1046	133	133.16	126.47	8	7.99	0.83692	-0.15	0.00	0.00	0.00
1488	117	124.36	120.75	5	4.79	0.75293	-7.15	0.45	0.44	0.01
1489	139	95.73	90.25	5	7.48	0.73298	+40.79	18.07	19.52	0.82
1491	0	25.98	25.98	0	0.00	0.81798	-25.98	51.97	25.94	0.00
1492	128	95.56	88.35	7	9.45	0.76369	+29.99	10.64	11.03	0.63
1493	114	126.12	120.51	8	7.46	0.75292	-11.58	1.24	1.17	0.04
2631	59	64.93	59.86	9	8.54	0.59393	-5.47	0.58	0.54	0.03
2632	59	44.46	39.52	7	8.62	0.57361	+12.93	4.63	4.76	0.30
2636	0	25.97	25.97	0	0.00	0.62603	-25.97	51.93	25.97	0.00
2637	58	51.56	47.61	6	6.49	0.60775	+5.95	0.81	0.80	0.04
2638	57	63.70	61.45	4	3.76	0.59725	-6.46	0.74	0.70	0.01

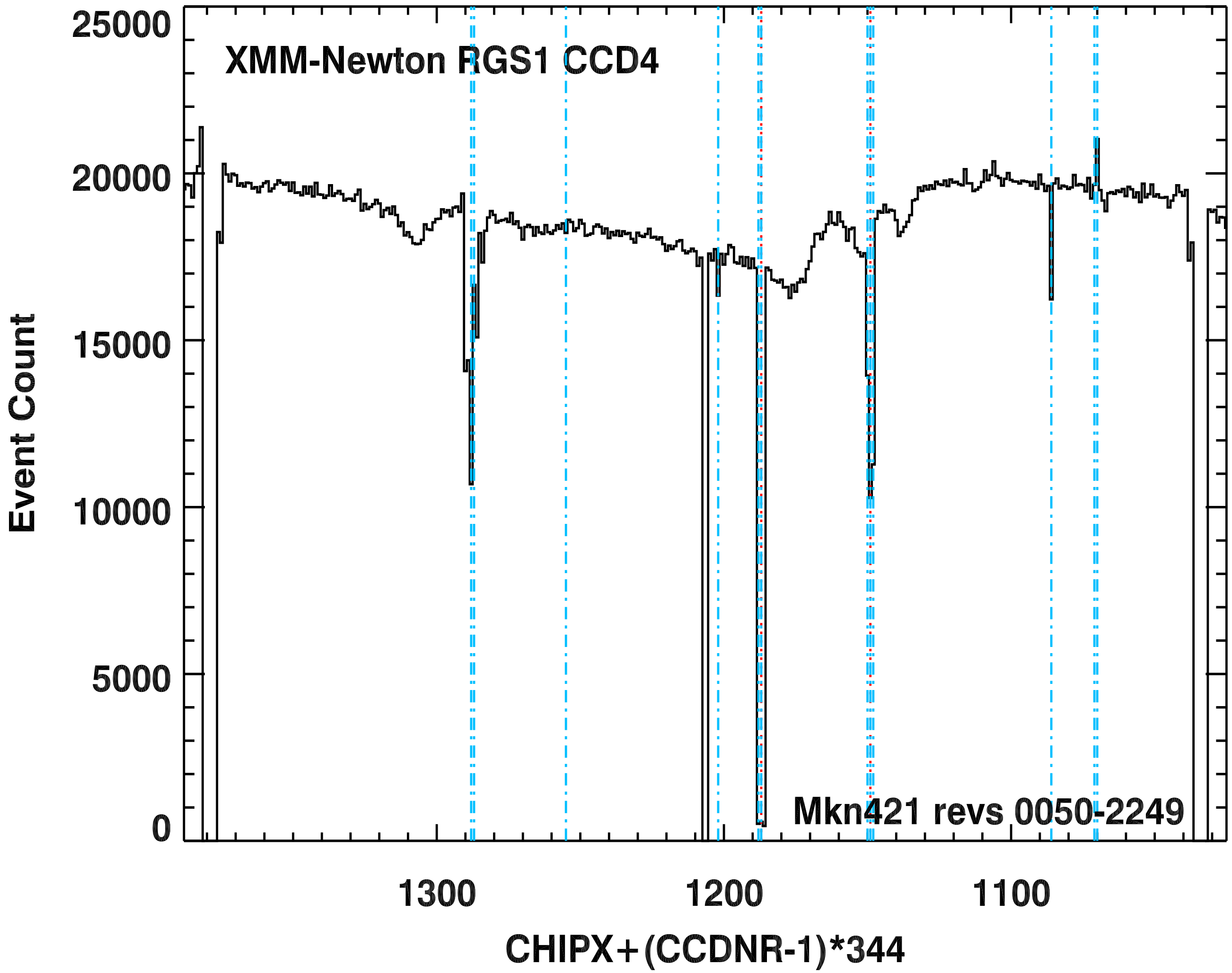
RGS statistics of empty channels

PKS2155-304 1349_0411780201 RGS1 1st order

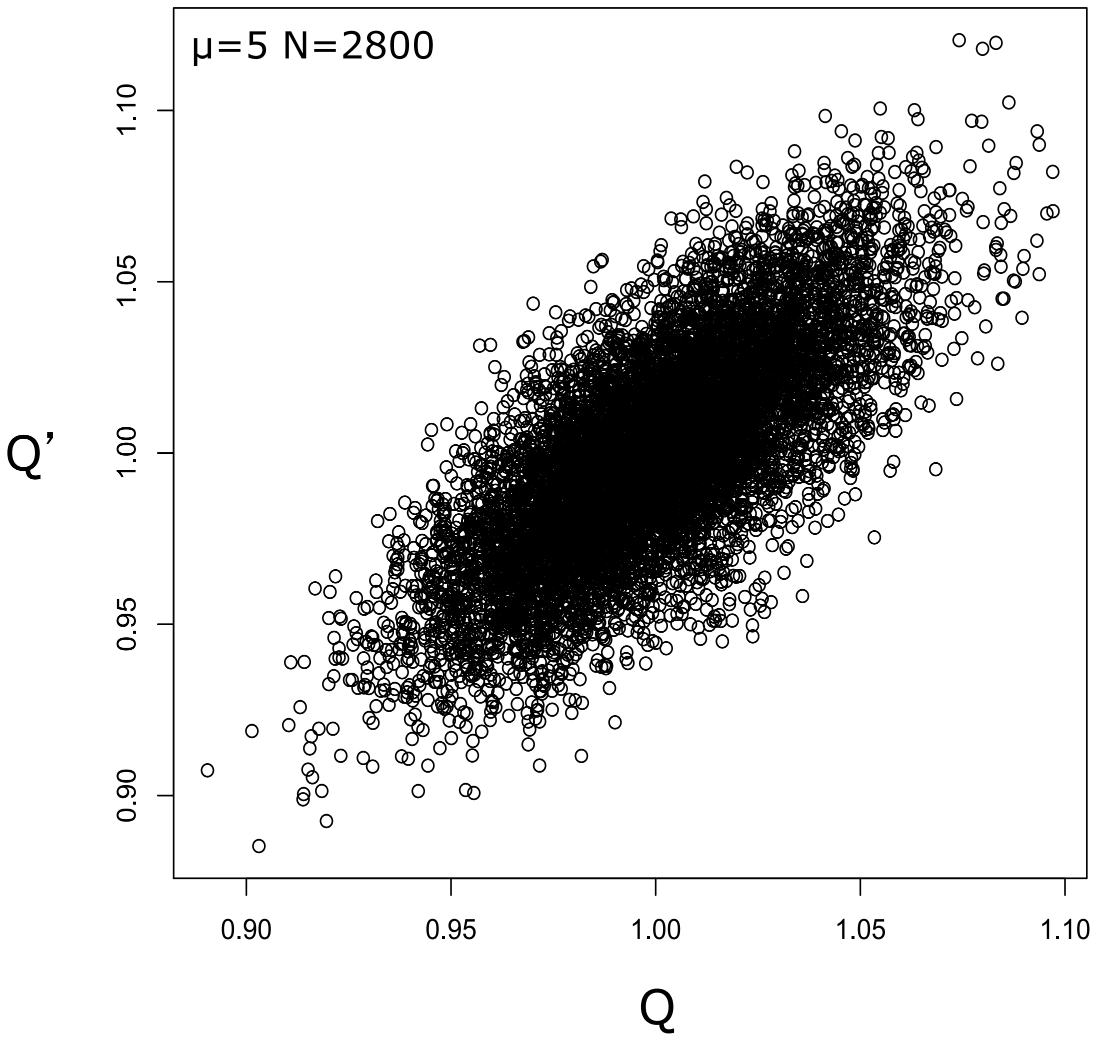
chan	nT	mT	mS	nB	mB	f	resid	dC	qT	qB
1044	0	1.81	1.81	0	0.00	0.93844	-1.81	3.62	1.81	0.00
1136	0	2.40	2.40	0	0.00	0.91035	-2.40	4.80	2.40	0.00
1148	0	10.74	10.74	0	0.00	0.90799	-10.74	21.48	10.71	0.00
1219	0	1.03	1.03	0	0.00	0.88172	-1.03	2.05	1.03	0.00
1491	0	25.98	25.98	0	0.00	0.81798	-25.98	51.97	25.94	0.00
1895	0	4.97	4.97	0	0.00	0.69172	-4.97	9.93	4.97	0.00
1940	0	17.77	17.77	0	0.00	0.69181	-17.77	35.55	17.77	0.00
2030	0	5.47	5.47	0	0.00	0.68609	-5.47	10.95	5.47	0.00
2188	0	13.80	13.80	0	0.00	0.74949	-13.80	27.60	13.80	0.00
2265	0	9.92	9.92	0	0.00	0.76517	-9.92	19.84	9.92	0.00
2636	0	25.97	25.97	0	0.00	0.62603	-25.97	51.93	25.97	0.00
2719	0	5.45	5.45	0	0.00	0.63750	-5.45	10.90	5.45	0.00
3183	0	14.42	14.42	0	0.00	0.54121	-14.42	28.84	14.42	0.00
3394	0	7.04	7.04	0	0.00	0.67660	-7.04	14.08	7.04	0.00

Fluctuations between neighbouring pixels

$Q=(n-\mu)^2/\mu \rightarrow Q'=(n_1-n_2)^2/(n_1+n_2)$



Comparing $Q=(n-\mu)^2/\mu$ & $Q'=(n_1-n_2)^2/(n_1+n_2)$



Fuzzy thinking

- Cherish data
- Be aware of the strengths and limitations of each instrument
- Make lots of plots
  - Model space
    - $\ln L(\theta)$
  - Data space
    - Data & model
  - Beware log-log plots
- Think about parameter independence
- 1σ errors always
  - Same for upper limits
- Make every decision a statistical decision
- Discard "aperture" methods
  - Optimum $\mu(x,y,t,\lambda)$ for all x,y,t,λ
- Make the best model possible
  - If there are 100 sources and 6 different backgrounds in your data
  - put 100 sources and 6 different backgrounds in your model

Ten commandments of data analysis

1. Don't rebin
2. n=0 is a perfectly good measurement
3. Don't subtract from the data, add to the model
4. Use Poisson statistics to explore parameter space
5. Report unreduced C-statistic, NBINS & NDOF (and NFREE/NPAR)
6. Report maximum-likelihood parameter estimates and $\Delta C=1$ errors
7. Calculate the goodness-of-fit Q-statistic
8. $\mu=0.\pm\sigma$ is a perfectly good parameter estimate
9. Beware of systematic errors
10. Beware of pile-up, event reconstruction and PI redistribution

Make every photon count.
Understand every photon and every bin.