The speed of a star image in the Gaia field of view from general attitude motion or scanning law

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Abstract. This note gives analytical expressions for the apparent speed along and across scan of a star in the Astro or Spectro field of view, as produced by the general attitude motion or (specifically) the Nominal Scanning Law (NSL). It is noted that the scanning law induces field-dependent variations in the along-scan image speed with an amplitude of 1.1 mas s^{-1} in addition to the well-known (and much larger) across-scan motion. The differential along-scan speed (Astro-1 *minus* Astro-2) has an amplitude of up to 1.7 mas s^{-1}. Version 3 clarifies some approximations made in the expressions for the NSL, and considers also the (non-nominal) reversed precession mode.

1 Introduction

The Nominal Scanning Law (NSL) of Gaia is defined such that star images will have approximately constant apparent speed in the along-scan (AL) direction of the Astro or Spectro field. It is well known that the precessional motion implied by the NSL also produces an across-scan (AC) motion of the images with an amplitude of up to ± 174 mas s^{-1}. It is less well known that the NSL also implies a (much smaller) variation of the AL image speed that should be taken into account e.g. for the windowing and which adds to the TDI error budget. In this note some general expressions for the image speed are derived.

2 Notations and assumptions

The attitude of Gaia specifies the celestial orientation of the Scanning Reference System (SRS) as function of time. The SRS is defined in the Gaia conventions document (GAIA–ARI–BAS–003) and Fig. 2 below. It is given by the orthogonal unit vectors $[\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}]$ with $\boldsymbol{x}$ bisecting the two Astro viewing directions and $\boldsymbol{z}$ normal to the viewing directions. In the SRS we may define spherical coordinates (φ, ζ) as in the figure.[1] The usual AL field angle η is related to φ through

$$\eta = \varphi - \psi = \varphi \mp \gamma/2 \qquad (1)$$

where $\psi = \pm\gamma/2$ is the location of the field centre relative the $\boldsymbol{x}$ axis. We use the convention that upper/lower sign signifies the preceding/following field of view (Astro-1/2).

[1] The angle φ is not used in the conventions document and therefore is not a standard notation.

Let $\boldsymbol{r}$ be the instantaneous proper direction to a star. For the present investigation we assume that this direction is fixed in the celestial reference system. Its instantaneous components in the SRS are given by

$$\left.\begin{aligned} \boldsymbol{x}'\boldsymbol{r} &= \cos\zeta\cos\varphi \\ \boldsymbol{y}'\boldsymbol{r} &= \cos\zeta\sin\varphi \\ \boldsymbol{z}'\boldsymbol{r} &= \sin\zeta \end{aligned}\right\} \tag{2}$$

where the prime ($'$) denotes the scalar product of two vectors (or the transpose of the preceding matrix). An equivalent expression is

$$\boldsymbol{r} = \boldsymbol{x}\cos\zeta\cos\varphi + \boldsymbol{y}\cos\zeta\sin\varphi + \boldsymbol{z}\sin\zeta \tag{3}$$

The instantaneous inertial angular velocity of the instrument is denoted $\boldsymbol{\omega}$. The components of $\boldsymbol{\omega}$ on the three instrument axes are denoted $(\omega_x, \omega_y, \omega_z)$; thus

$$\boldsymbol{\omega} = \boldsymbol{x}\omega_x + \boldsymbol{y}\omega_y + \boldsymbol{z}\omega_z \tag{4}$$

According to the NSL ω_z is constant and equal to 60 arcsec s^{-1}, while the precession rate, given by $|\dot{\boldsymbol{z}}| \equiv (\omega_x^2 + \omega_y^2)^{1/2}$, is also approximately constant with a maximum value of 0.174 arcsec s^{-1}. The direction of $\dot{\boldsymbol{z}}$ varies over the spin period such that (ω_x, ω_y) is approximately normal to a circle from $\boldsymbol{z}$ to the sun. Defining the spin phase Ω in the usual way (e.g. SAG–LL–030), and ignoring the component of $\boldsymbol{\omega}$ that originates in the annual motion of the nominal sun, we have for the NSL:

$$\left.\begin{aligned} \omega_x &\overset{\text{NSL}}{\simeq} +C\cos\Omega \\ \omega_y &\overset{\text{NSL}}{\simeq} -C\sin\Omega \end{aligned}\right\} \tag{5}$$

where $C = +|\dot{\boldsymbol{z}}|$ in the (nominal) forward precession mode of the NSL ($\dot{\nu} > 0$), and $C = -|\dot{\boldsymbol{z}}|$ in the reversed precession mode ($\dot{\nu} < 0$). Thus, both ω_x and ω_y vary sinusoidally over the spin period (6 hours) with an amplitude of up to 0.174 arcsec s^{-1}.

We seek expressions for the absolute AL and AC angular rates $\dot{\eta} = \dot{\varphi}$ and $\dot{\zeta}$ of the star in the field, and also expressions for the differential rates between the two fields of view.

3 Absolute rates of the field angles

Since $\boldsymbol{r}$ is fixed while $[\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}]$ rotates with angular velocity $\boldsymbol{\omega}$ we have

$$\left.\begin{aligned} \dot{\boldsymbol{r}} &= \boldsymbol{0} \\ \dot{\boldsymbol{x}} &= \boldsymbol{\omega}\times\boldsymbol{x} \\ \dot{\boldsymbol{y}} &= \boldsymbol{\omega}\times\boldsymbol{y} \\ \dot{\boldsymbol{z}} &= \boldsymbol{\omega}\times\boldsymbol{z} \end{aligned}\right\} \tag{6}$$

Taking the time derivative of the left member of (2a) gives

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{x}'\boldsymbol{r}) = \dot{\boldsymbol{x}}'\boldsymbol{r} + \boldsymbol{x}'\dot{\boldsymbol{r}} = (\boldsymbol{\omega}\times\boldsymbol{x})'\boldsymbol{r} = (\boldsymbol{x}\times\boldsymbol{r})'\boldsymbol{\omega} \tag{7}$$

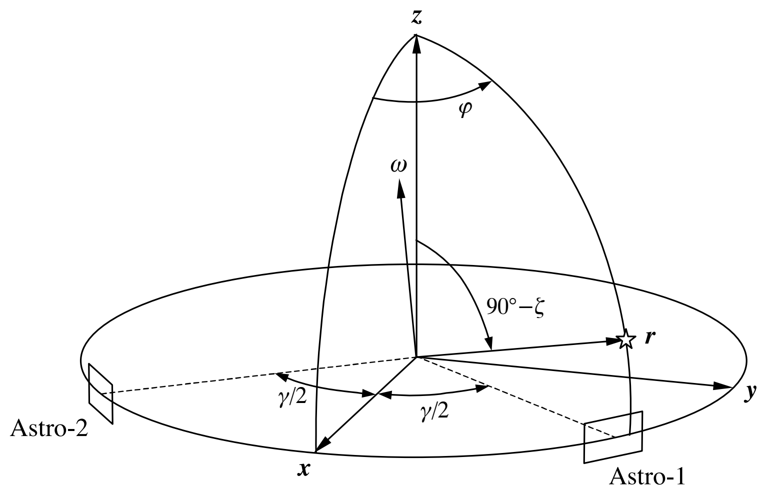

FIGURE 1: Definition of the Scanning Reference System $[\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}]$ and the corresponding spherical angles (φ, ζ) for the instantaneous direction to a star at $\boldsymbol{r}$. γ is the basic angle, $\boldsymbol{\omega}$ the instantaneous inertial angular velocity of the instrument.

where we have used that $(\boldsymbol{a} \times \boldsymbol{b})'\boldsymbol{c} = (\boldsymbol{b} \times \boldsymbol{c})'\boldsymbol{a} = (\boldsymbol{c} \times \boldsymbol{a})'\boldsymbol{b}$. From (3) we have

$$\boldsymbol{x} \times \boldsymbol{r} = (\boldsymbol{x} \times \boldsymbol{x}) \cos\zeta \cos\varphi + (\boldsymbol{x} \times \boldsymbol{y}) \cos\zeta \sin\varphi + (\boldsymbol{x} \times \boldsymbol{z}) \sin\zeta = \boldsymbol{z} \cos\zeta \sin\varphi - \boldsymbol{y} \sin\zeta \quad (8)$$

so that

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{x}'\boldsymbol{r}) = +\,\omega_z \cos\zeta \sin\varphi - \omega_y \sin\zeta \quad (9)$$

where we have put $\boldsymbol{z}'\boldsymbol{\omega} = \omega_z$, etc. Taking the time derivative of the right member of (2a), identifying with the above expression, and carrying out the corresponding manipulations for (2b) and (2c), we find

$$\left.\begin{aligned} -\,\omega_y \sin\zeta \qquad\quad & +\,\omega_z \cos\zeta \sin\varphi = -\,\dot\varphi \cos\zeta \sin\varphi - \dot\zeta \sin\zeta \cos\varphi \\ +\,\omega_x \sin\zeta \qquad\quad & -\,\omega_z \cos\zeta \cos\varphi = +\,\dot\varphi \cos\zeta \cos\varphi - \dot\zeta \sin\zeta \sin\varphi \\ -\,\omega_x \cos\zeta \sin\varphi & +\,\omega_y \cos\zeta \cos\varphi = \qquad\qquad\qquad\quad +\,\dot\zeta \cos\zeta \end{aligned}\right\} \quad (10)$$

From the last equation (10c) we immediately have

$$\dot\zeta = -\,\omega_x \sin\varphi + \omega_y \cos\varphi \overset{\text{NSL}}{\simeq} -C \sin(\Omega + \varphi) \quad (11)$$

Note that the first equality holds for any attitude, while the second assumes the approximate NSL relations in (5). Note also that $\Omega + \varphi$ is simply the abscissa of the star relative to that of the sun.

To obtain $\dot\eta = \dot\varphi$ we multiply (10a) by $-\sin\varphi$, (10b) by $\cos\varphi$ and add the equations; the result after division by $\cos\zeta$ is

$$\dot\eta = -\,\omega_z + (\omega_x \cos\varphi + \omega_y \sin\varphi) \tan\zeta \overset{\text{NSL}}{\simeq} -\,\omega_z + C \cos(\Omega + \varphi) \tan\zeta \quad (12)$$

Averaged over time (or Ω) it is seen that, for any point in the field, $\langle\dot\zeta\rangle = 0$ and $\langle\dot\eta\rangle = -\omega_z$, as expected. This shows that the pixel columns should be aligned with $\zeta = \text{const}$ and the pixel lines equidistant in η in order to minimize tracking errors.

4 Relation to the attitude

The standard way to parameterize the attitude is in terms of the continuous quaternion function $\boldsymbol{q}(t)$. For the attitude rates we also need the time derivative $\dot{\boldsymbol{q}}(t)$. Equation (11) in SAG–LL–035 gives a relation between these quantities and the spin vector $\boldsymbol{\omega}$ expressed in the SRS. The inverse of that relation is readily obtained as

$$\left.\begin{aligned} \omega_x &= 2(+\, q_4\dot{q}_1 + q_3\dot{q}_2 - q_2\dot{q}_3 - q_1\dot{q}_4) \\ \omega_y &= 2(-\, q_3\dot{q}_1 + q_4\dot{q}_2 + q_1\dot{q}_3 - q_2\dot{q}_4) \\ \omega_z &= 2(+\, q_2\dot{q}_1 - q_1\dot{q}_2 + q_4\dot{q}_3 - q_3\dot{q}_4) \end{aligned}\right\} \tag{13}$$

Together with (11) and (12) this completely specifies the rates of the field angles as function of the general attitude and the field angles.

5 Approximate absolute rates

In order to write the rates in terms of the field angles (η, ζ) we use (1) and substitute $\varphi = \eta \pm \gamma/2$ in (11) and (12). However, in order to see the main effects it is sufficient to put $\varphi \simeq \pm\gamma/2$, which gives the approximate formulae:

$$\left.\begin{aligned} \dot{\eta} &\simeq -\,\omega_z + \big[\omega_x \cos(\gamma/2) \pm \omega_y \sin(\gamma/2)\big] \tan\zeta \\ \dot{\zeta} &\simeq \mp\,\omega_x \sin(\gamma/2) + \omega_y \cos(\gamma/2) \end{aligned}\right\} \tag{14}$$

These expressions are correct to $O(\eta\zeta\omega_{x,y})$ in $\dot{\eta}$ and to $O(\eta\omega_{x,y})$ in $\dot{\zeta}$, which is sufficient e.g. for windowing.

The amplitude of $\dot{\zeta}$ is the same as that of ω_x and ω_y, i.e. up to 174 mas s^{-1}. The deviation of $\dot{\eta}$ from $-\omega_z$ has a much smaller amplitude thanks to the $\tan\zeta$ factor. For a central field transit ($\zeta = 0$) it vanishes, but at the extreme edges ($|\zeta| \simeq 0.36°$) it may have an amplitude of about 1.1 mas s^{-1}.

6 Approximate differential rates

The difference in rate between Astro-1 and Astro-2 is of interest for the windowing strategy. From (14) we find the differential rates

$$\left.\begin{aligned} \Delta\dot{\eta} \equiv \dot{\eta}_{\mathrm{Astro1}} - \dot{\eta}_{\mathrm{Astro2}} &\simeq +2\omega_y \sin(\gamma/2)\tan\zeta \overset{\mathrm{NSL}}{\simeq} -2C\sin(\gamma/2)\sin\Omega\tan\zeta \\ \Delta\dot{\zeta} \equiv \dot{\zeta}_{\mathrm{Astro1}} - \dot{\zeta}_{\mathrm{Astro2}} &\simeq -2\omega_x \sin(\gamma/2) \overset{\mathrm{NSL}}{\simeq} -2C\sin(\gamma/2)\cos\Omega \end{aligned}\right\} \tag{15}$$

$\Delta\dot{\zeta}$ has an amplitude of up to 265 mas s^{-1}. $\Delta\dot{\eta}$ is zero at $\zeta = 0$ and has a maximum amplitude of about 1.7 mas s^{-1} at the extreme edges ($|\zeta| \simeq 0.36°$). The maximum amplitude of $\Delta\dot{\eta}$ is reached at the same time as $\Delta\dot{\zeta} = 0$, and vice versa.