# Calibration constraints in IDU


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## Abstract


This TN specifies the constraints that the LSF/PSF calibration in IDU should respect in order to avoid degeneracies with the AGIS calibration.

# Document History

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# Contents

# 1 Introduction

This TN specifies the constraints that the LSF/PSF calibration in IDU should respect in order to avoid degeneracies with the AGIS calibration.

# 2 Background

This section outlines the general relations between AGIS and IDU, and in particular it explains why the AGIS calibration and LSF/PSF calibrations (Fig. 1) need to be considered together. It is provided for context only and is not essential for actually implementing the constraints.

The AGIS calibration should ultimately be purely geometric, i.e. independent of colour and magnitude. In particular, centroid displacements depending on colour must be completely taken out by the corresponding calibration in IDU (Fig. 2). However, until that goal is achieved, AGIS will contain chromatic calibration terms in order to improve the quality of the solution and to cope with sources without CU5 colour.

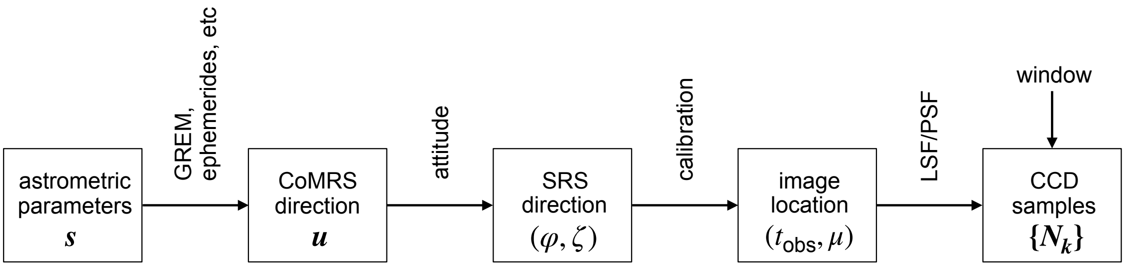


FIGURE 1: Main steps in the forward modelling of the CCD samples of a point source. For simplicity the photometric part of the modelling is left out, and a number of additional dependencies are represented by the 'window' model.

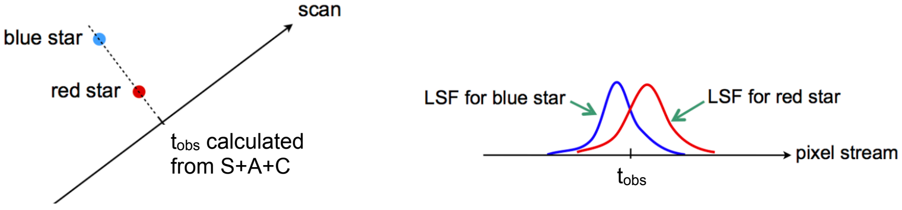


FIGURE 2: Why the AGIS calibration must (ultimately) not depend on colour and magnitude. *Left:* Consistent modelling of complex objects such as binaries is only possible if the calculated location ($t_{\text{obs}}$) of a point source is independent of its colour and magnitude. *Right:* This requires that chromatic shifts are removed by the LSF calibration.

# 3 Definitions

We describe hereafter mainly the AL relationships and parametrisation of the LSF, but similar considerations apply to the AC coordinate for the PSF (see Sect. 6).

The terms ‘centroid’, ‘location’, etc. are used with sometimes slightly different meanings. For this TN it is important to be absolutely clear what is meant. The relevant concepts and terms are explained in Fig. 3. For a point source, the location $\kappa$ is converted to $t_{\text{obs}}$ by referring back to the central exposure time (fiducial line) and transforming to TCB by means of the time ephemeris.

Except for symmetric LSFs, the definition of the centroid (‘mid-point’ of the LSF) is non-trivial and purely a matter of convention. Possible conventions include:

- the centre of gravity (not a good choice, as it is very sensitive to the wings);
- the median (better, but still needs the wings);
- Tukey’s biweight (Press et al., 2007) or some similar finite-width local $M$-estimator (even better);
- the centroid implicit in the choice of basis functions (to be explained).

*The calibrations will depend on this choice* – and it is possible that some choices result in simpler or more accurate calibrations than others, for example in terms of linearity with $\nu_{\text{eff}}$. Such considerations are beyond the scope of this TN (cf. Lindegren LL-064 and Lindegren LL-068).

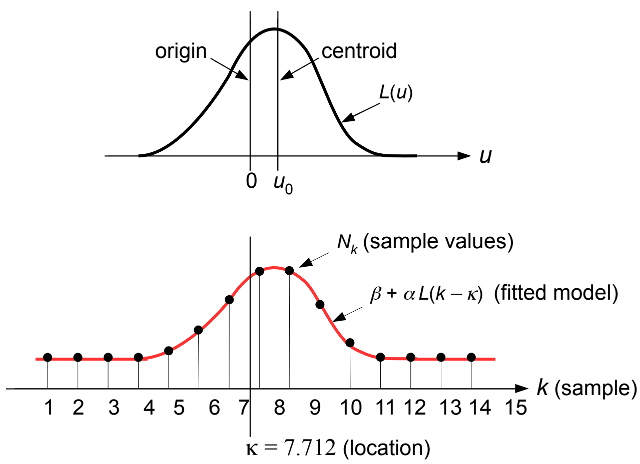


FIGURE 3: Definition of centroid, origin and location: the top diagram shows a schematic LSF, $L(u)$, with the origin ($u = 0$) and centroid ($u = u_0$) indicated. The bottom diagram shows the location ($\kappa$) of the LSF in the stream of sample values $N_k$. In this case the window model consists of a uniform background of brightness $\beta$ (in electrons per sample) plus a single point source of intensity $\alpha$ (in electrons) at $k = \kappa$. Adapted from Lindegren (LL-080).

# 4 Parametrisation of the LSF

## 4.1 Shift and shape parameters

The general LSF model proposed in Lindegren (LL-088) is

$$L(u) = H_0(u - h_0) + \sum_{n=1}^{N-1} h_n\, H_n(u - h_0)\,, \tag{1}$$

where $H_n(x)$, $n = 0 \dots N-1$ are basis functions and $h_n$ the parameters, and $u$ the AL coordinate in pixels (TDI periods). To ensure that $\int_{-\infty}^{\infty} L(u)\,\mathrm{d}u = 1$, the basis functions must satisfy

$$\int_{-\infty}^{\infty} H_n(u)\,\mathrm{d}u = \delta_{n0}\,, \quad n = 0 \dots N-1\,. \tag{2}$$

If $H_0(u)$ is close to the mean LSF, and the variation of LSFs is not too large, then the sum in (1) will be a relatively small correction to the first term. It is advantageous, but not necessary, that the basis functions are mutually orthogonal.[1]

We may call $h_0$ the *shift parameter* and $h_n$ ($n > 0$) the *shape parameters* of the LSF model. The unit for $h_0$ is the pixel (TDI period), while $h_n$ ($n > 0$) are dimensionless with a scaling that depends on how the basis functions (for $n > 0$) are defined.

The shift parameter is needed for two reasons:

- it eliminates the need to represent arbitrary (albeit small) shifts by means of the basis functions, which reduces the required dimensionality ($N$);
- it allows the calibrated LSF (represented by $h_0$, $h_1$, ..., $h_{N-1}$) to be shifted very easily without changing the shape – this is needed as explained in Sect. 5.

However, the introduction of $h_0$ is a major complication for CALIPD because it is non-linear, that is $\partial^2 L/\partial h_0^2 \neq 0$, which means that the fitting must be iterated.[2] The following linear model has therefore been proposed as an alternative to (1):

$$L(u) = H_0(u) - h_0\, H_0'(u) + \sum_{n=1}^{N-1} h_n\, H_n(u)\,. \tag{3}$$

Equation (3) is derived from (1) by a Taylor expansion, ignoring terms of order $O(h_0 h_n)$, $n \geq 0$.

[1] That is, $\int_{-\infty}^{\infty} H_m(u) H_n(u) w(u)\,\mathrm{d}u = \delta_{mn}$ for $m \neq n$, where $w(u)$ is some weight function; typically $w(u) = 1$ is used. The fitting of (1) to actual data will not give uncorrelated parameters even for orthogonal basis functions, because the fitting normally uses a different weight function than this $w(u)$.

[2] This would essentially use Eq. (4).

For future reference we note that a more accurate linear representation may be obtained by expanding (1) around a provisional value of the shift parameter, $\tilde{h}_0$:

$$L(u) = H_0(u - \tilde{h}_0) - \delta h_0\, H_0'(u - \tilde{h}_0) + \sum_{n=1}^{N-1} h_n\, H_n(u - \tilde{h}_0)\,. \tag{4}$$

The final shift parameter is then $h_0 = \tilde{h}_0 + \delta h_0$. This is (potentially) more accurate than (3), because $\delta h_0$ can be made arbitrarily small by means of a proper choice of $\tilde{h}_0$.

The use of (3) instead of (1) has one important consequence. With (1) the shape of the LSF is strictly preserved when the model is shifted by changing $h_0$ (for example, setting a non-zero $h_0$ to zero). This is no longer the case for (3): changing $h_0$ shifts the LSF, but it also entails a (small) change of its shape, proportional to $H_0'(u)$. This means that the resulting LSF is not exactly the same as would have been obtained by fitting the model to the shifted data. However, if $H_n(u)$ (for $n > 0$) are orthogonal to $H_0'(u)$, then it can be assumed that the fitted shape parameters are practically independent of the shift.[3]

## 4.2 Dependencies

The LSF model has a number of dependencies that may be represented by a vector $\boldsymbol{x}$. Symbolically the LSF can thus be written $L(u|\boldsymbol{x})$, which in practice means that the LSF parameters are functions of $\boldsymbol{x}$, i.e. $h_n(\boldsymbol{x})$ for $n = 0 \dots N-1$.

The components of $\boldsymbol{x}$ could be the following variables, or a subset of them:

- the time $t$;
- the field of view index $f$;
- the CCD row and strip indices $rs$;
- the gClass $w$;
- the gate $g$;
- the CCD column index $\mu$ (or the stitch block index $b$);
- the AC rate $\dot{\mu}$;
- the colour of the source $c$ (specifically, the effective wavenumber $\nu_{\text{eff}}$).

The actual representations used for $h_n(\boldsymbol{x})$ need not concern us here, although we return to this question in Sect. 7.1.

It is assumed that the dependences on the AC rate $\dot{\mu}$ are not empirically calibrated but the expected effects are applied analytically (top-hat convolution).[4] Although the LSF and PSF thus depend on $\dot{\mu}$, that variable can effectively be ignored for the rest of this TN.

[3] Referring to footnote 1, this orthogonality condition should ideally use a weight function $w(u)$ close to the typical weights in IPD, which are inversely proportional to the variances of the CCD samples (TBC).

[4] $\dot{\mu}$ is the time derivative of the instantaneous AC coordinate $\mu$ during the CCD integration. It is approximately equal to the time derivative of the AC field angle $\zeta$ (Lindegren, GAIA-LL-056), converted to pixels, but the precise value contains an additional correction for the non-alignment of the CCD columns with the field angles.

In the following it is convenient to partition the vector $\boldsymbol{x}$ as

$$\boldsymbol{x} = [t,\ \boldsymbol{x}_{\rm nc},\ \boldsymbol{x}_{\rm c}]\,, \tag{5}$$

where the chromatic part $\boldsymbol{x}_{\rm c}$ currently contains only one variable, namely $c = \nu_{\rm eff}$, while the non-chromatic part $\boldsymbol{x}_{\rm nc}$ contains all the rest except $t$.[5]

## 5 Constraints on the LSF calibration

Both the AGIS calibration and the IDU calibration depend on all the variables listed above (Sect. 4.2). What concerns us here is the overlap between the two calibrations, i.e. where both could potentially model the same physical effect. Clearly such an overlap only exists for the shift parameter $h_0(\boldsymbol{x})$, since the shape parameters are not calibrated in AGIS.

For the overlap, constraints are needed to ensure that the combined AGIS+IDU calibration does not introduce unwanted astrometric biases, such as a dependence of the reference frame on the gClass or colour.

Such constraints are already implemented in the AGIS part of the calibration (Lindegren & Bombrun, LL-121). At AGIS-29 (Barcelona, 28–29 May 2019) I concluded that the same constraints must be applied to the LSF shift parameters. However, this is technically complicated, and impractical because the constraints may in fact change as the AGIS calibration develops.

A much simpler idea is presented here and detailed in Sect. 5.1 and 5.2: just ensure that the IDU part of the overlap calibration remains exactly zero. Then the total AGIS+IDU calibration will automatically satisfy the constraints as soon as the AGIS constraints are in place, and the AGIS constraints may be modified without affecting the IDU.

Effectively, this means that a clear separation is made between on one hand the effects that *must* be calibrated in IDU, and on the other the remaining effects that could also be calibrated in AGIS. IDU should *only* take care of the calibrations of the first kind, and AGIS all the rest.

So which effects and dependencies must and should be calibrated in IDU? Clearly all the LSF shape parameters, i.e. $h_n(\boldsymbol{x})$ for $n > 0$, and their dependences on the variables listed in Sect. 4.2, belong to this category.

For the LSF shift parameter $h_0(\boldsymbol{x})$, only the colour-dependent part ($h_{01}$ in Eq. 6) must be calibrated in IDU. The reason for this is explained in Sect. 2 and Fig. 2.

It follows that no constraints are needed on the LSF shape parameters ($n > 0$), while for the shift parameter $h_0$ there is an overlap with AGIS that may require constraints.

[5] If, in the future, the calibrations will depend on $G$, then this variable will also be included in $\boldsymbol{x}_{\rm c}$.

## 5.1 Non-chromatic calibration

As discussed in Sect. 7.1, the chromatic part of $h_0$ must currently (Cycle 4) depend linearly on $c = \nu_{\text{eff}}$. On the other hand, the chromaticity itself, i.e. the slope of displacement versus colour, may depend on any of the other variables. The most general expression for the shift parameter is therefore

$$h_0(\boldsymbol{x}) = h_{00}(t, \boldsymbol{x}_{\text{nc}}) + h_{01}(t, \boldsymbol{x}_{\text{nc}}) \times \left(c - c^{\text{ref}}\right) \,, \tag{6}$$

where $c^{\text{ref}}$ is the adopted reference colour. In principle $c^{\text{ref}}$ could be a function of $\boldsymbol{x}_{\text{nc}}$, but that is probably not needed and a fixed value will be used ($\nu_{\text{eff}} = 1.43~\mu\text{m}^{-1}$ exactly[6]).

The relevant constraint for the non-chromatic LSF calibration is consequently

$$h_{00}(t, \boldsymbol{x}_{\text{nc}}) = \boldsymbol{0}\,. \tag{7}$$

That is, at the reference colour there is at no time any dependence of the shift parameter on any of the non-chromatic variables.

In practice this constraint could be implemented as a two-step procedure:

1. First, make a calibration of $h_0$ as a function of all the relevant variables in $\boldsymbol{x}$. This step could use the 'old' (pre-Cycle 4) algorithms after adding the shift parameter to the unknowns according to (1) or (3). This calibration determines the two functions $h_{00}(t, \boldsymbol{x}_{\text{nc}})$ and $h_{01}(t, \boldsymbol{x}_{\text{nc}})$ in (6).
2. Then set $h_{00}(t, \boldsymbol{x}_{\text{nc}})$ to zero. To the extent that the determination of the shape parameters are invariant to a pure shift of the samples (see discussion in Sect. 4.1), this is equivalent to fitting $h_{01}(t, \boldsymbol{x}_{\text{nc}})$ subject to the constraint (7).

## 5.2 Chromatic calibration

The constraint (7) ensures that the LSF calibration does not introduce any AL image shift that can be represented as a linear combination of the six psi functions $\psi_j(t)$.[7] This is necessary in order that AGIS should be able to construct a consistent reference frame across all magnitudes (or gClasses/gates), aligned with and non-rotating with respect to the ICRS via the quasars.

However, from (7) alone this is only ensured for sources that have the reference colour $c^{\text{ref}}$. It is entirely possible for redder or bluer sources to obtain a different reference frame, namely if $h_{01}(t, \boldsymbol{x}_{\text{nc}}) \times \left(c - c^{\text{ref}}\right)$ has a time-dependent component in the psi-space. Thus, the following additional constraints need to be imposed on $h_{01}(t, \boldsymbol{x}_{\text{nc}})$:

$$\forall w, g: \quad \int \bar{h}_{01}(t, w, g)\psi_j(t)\, \text{d}t = 0\,, \quad j = 1\ldots 6\,, \tag{8}$$

[6] See Jira C3ACT-177

[7] See Sect. 3.3.1 in Lindegren & Bombrun LL-121. The psi functions span a set of time-dependent AL image shifts corresponding to the six degrees of freedom of the reference frame – the 'psi space'.

where

$$\bar{h}_{01}(t, w, g) = \langle h_{01}(t, \boldsymbol{x}_{\rm nc})\rangle \tag{9}$$

is the average of $h_{01}$ over $f$, $rs$, and $\mu$ (or $b$). The integral in (8) is taken over the time interval(s) used by AGIS. The number of constraints is six times the cardinality of $wg$.

In practice this constraint can be implemented by first calibrating $h_{01}(t, \boldsymbol{x}_{\rm nc})$ in the usual way, and then subtracting its projection on the psi-space.

It is not obvious if this constraint should be implemented in IDU or AGIS. As is clear from the above, it relies on certain information more readily available in AGIS (the psi functions, the relevant time intervals, etc.). An alternative solution, recommended here, is that IDU only provides the time-dependent functions $\bar{h}_{01}(t, w, g)$, while AGIS computes the required corrections in the AGIS pre-processor and incorporates them as a fixed correction in the AGIS calibration (similar to the BAM corrections). This solution preserves as far as possible the autonomy of IDU and AGIS.

At this point we may ask the question: why not just remove the time dependence of $h_{01}$, in the same way as was done for the non-chromatic part? This is however not possible, because the actual chromaticity, which is of course time-dependent, must be part of the LSF calibration as explained before. The constraint (8) is in fact (only) meant to remove any spurious component of the chromaticity that was accidentally introduced, for example by a preceding AGIS solution having a chromatic reference frame.

The procedure outlined above is actually a bit dubious. In principle there is nothing to prevent the actual (physical) chromaticity of the instrument to have a non-zero component in the psi-space. If there is indeed such a component, it will be taken out by (8) and the physical effect will remain uncalibrated (and probably difficult to detect in AGIS except for the quasars). The result will be a chromatic reference frame. This will surely happen at some level, but as I can see no better solution we can only hope that it is small enough to be tolerable.

It is expected that the application of the chromatic constraints has a rather small impact on the astrometric solution. The reason for this is basically that the variation of $\boldsymbol{\omega}$ for the bright sources, as estimated from a comparison with HIPPARCOS positions, is quite small compared with the non-chromatic offset. It could then be acceptable to defer the implementation of (8) to a later stage in Cycle 3, when more experience has been gained on the overall impact of the many improvements in IDU.

# 6 Parametrisation of the PSF

The AC components of the PSF calibration are less critical than the AL components, but the non-chromatic constraints (7) are relevant here as well, and can to be applied as outlined in Sect. 5.1. The 2D version of (3) for the PSF is

$$L(u,v) = H_0(u,v) - h_0^{\parallel}\frac{\partial H_0}{\partial u}(u,v) - h_0^{\perp}\frac{\partial H_0}{\partial v}(u,v) + \sum_{n=1}^{N-1} h_n\, H_n(u,v)\,, \qquad (10)$$

where $u$ and $v$ are AL and AC pixel coordinates, $H_n(u,v)$ the 2D basis functions (shapelets) with $H_0(u,v)$ the mean PSF, and $h_0^{\parallel}$ and $h_0^{\perp}$ the AL and AC shift parameters.

If the AC shift parameter $\hbar_v$ is chromatic, one needs in principle additional constraints corresponding to (8). The AC psi functions are very different from their AL versions, and depend on $f$; thus no averaging over $f$ should be made in (9), and (8) will apply separately for each $f$. However, since the AC psi functions oscillate with a 6 h period, there is in fact no need for the chromatic constraints AC, provided that the AC chromaticity model does not allow significant variations on a 6 h time scale.

# 7 Other considerations

## 7.1 Harmonisation of models

A certain degree of harmonisation is desirable (even required?) between IDU and AGIS when it comes to the choice of model dependences and the variables to use. For example, if the LSF shape parameters are found to depend on some variable $X$, and are calibrated as functions of $X$, then it is very probable that $X$ must also be considered in the AGIS calibration model.

The chromaticity models require special attention. IDU and AGIS need to agree not only on the choice of reference colour ($c^{\mathrm{ref}}$), but also on the actual form of the dependence on $c$ for the shift parameter. Currently this is linear, as in Eq. (6), and it should remain so until there is a very clear need for something more sophisticated. (The shape parameters could have a more complicated [non-linear] dependence on $c$.)

For both the IDU and AGIS it is assumed that the linear dependence on $c$ in Eq. (6) holds in a certain range $c_{\mathrm{min}} \leq c \leq c_{\mathrm{max}}$. This range should be sufficient to contain all physically realistic colours. For $c = \nu_{\mathrm{eff}}$ it could be $[c_{\mathrm{min}}, c_{\mathrm{max}}] = [1.0, 2.0]\ \mu\mathrm{m}^{-1}$. No clamping of the colour should be made for values of $c$ outside this range, since such values are most likely wrong; instead the default colour ($c^{\mathrm{ref}}$) should be used.[8]

[8] See Jira C3ACT-236 and POACT-134 for further discussion on this issue, and IDU-514, IDU-541 for the implementation in IDU.

### 7.2 Soft reset

A reset of the IDU in terms of the LSF calibration and IPD has been initiated as a way to understand the origin and mechanisms of certain systematics observed in AGIS. It was suspected that systematics in the DR2 astrometry could be introduced in the IPD via the LSF calibration, reappear in subsequent AGIS solutions and thus cemented in the AGIS–IDU loop. The reset is designed to break this vicious circle by making the initial IPD completely independent of any previous astrometry. This can be achieved by using a simple, non-chromatic LSF, such as a Gaussian, for the initial IPD.

A drawback of this is that the IPD will be significantly less precise because the Gaussian is usually a bad fit to the data. In particular it will strongly bias the flux estimates.

Alternatively, a 'soft reset' could be made by zeroing all shift parameters in the LSF calibrations. This would give more precise IPD without introducing shifts that might depend on the previous astrometry; in particular the flux estimates should not be biased.

## 8 Testing the constraints

The implementation of the constraints described in Sect. 5 can be regarded as successful if the AstroElementaries (specifically $t_{\rm obs}$) resulting from a run of CALIPD (ELSF calibration) and IPD are insensitive to a certain kind of perturbation to the source parameters from AGIS. This 'singular perturbation' corresponds to a colour-dependent change of the celestial reference frame of the form[9]

$$\boldsymbol{\varepsilon}(t,c) = \boldsymbol{\varepsilon}_0 + (t - t^{\rm ref})\boldsymbol{\omega}_0 + (c - c^{\rm ref})\left(\boldsymbol{\varepsilon}_1 + (t - t^{\rm ref})\boldsymbol{\omega}_1\right) , \qquad (11)$$

which has 12 free parameters (the $X$, $Y$, $Z$ components of the vectors $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\omega}_i$, $i = 0,\ 1$).

A flowchart of the generic test is in Fig. 4. Proposed specifics of the test are given in Appendix A.

A practical complication is that the non-chromatic constraints (8), as discussed in Sect. 5.2, are not actually implemented in IDU, which would make a complete test non a self-contained part of IDU. The proposed solution is that only the non-chromatic constraints are rigorously tested, as described below, while for the chromatic constraints it is merely verified that the functions $\bar{h}_{01}(t,w,g)$ have the expected general characteristics.

[9]Singular perturbations can be defined also for the attitude, calibration and global parameters, but these would be equivalent (in terms of the computed field angle residuals) to a singular perturbation of the source parameters, so there is no need to use them in the test.

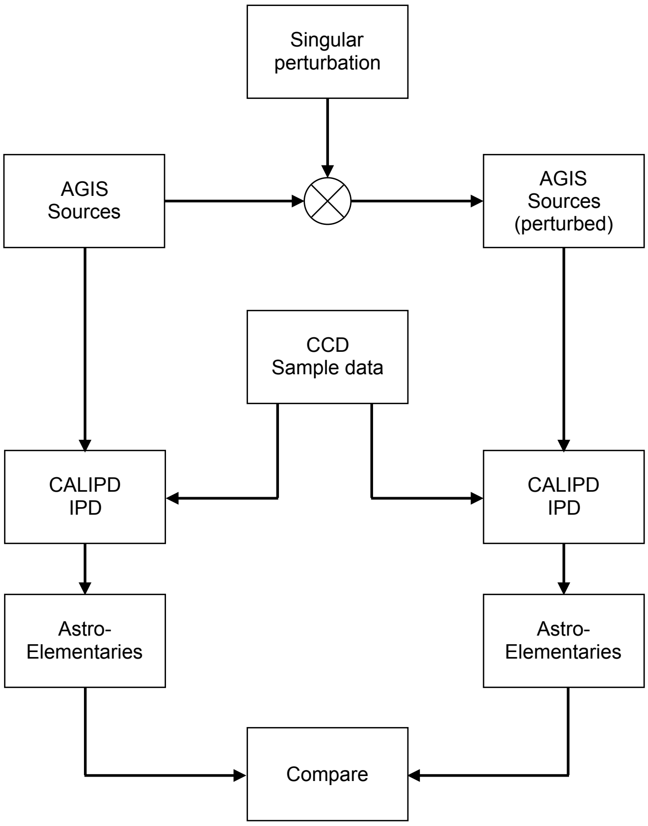


FIGURE 4: Flowchart for testing the implementation of the constraints in IDU.

## 8.1 Testing the non-chromatic constraints

Because the chromatic constraints are not applied in the test, the AstroElementaries from the two branches in Fig. 4 will only be equal for sources of the reference colour $c^{\text{ref}}$.

The test of the non-chromatic constraints should therefore check that $t_{\text{obs}}$ agree, to within a certain tolerance, for sources with small $|c - c^{\text{ref}}|$. See Appendix A for the choice of tolerances.

## 8.2 Testing the chromatic constraints

Sources with $c \neq c^{\text{ref}}$ will have a time-dependent difference in $t_{\text{obs}}$ from the two branches. The difference should be proportional to $c - c^{\text{ref}}$, and the constant of proportionality should be a linear combination of the psi functions, corresponding to the coefficients $\varepsilon_1$ and $\omega_1$.

The test could consist of a plot of $\Delta t_{\text{obs}}$ versus time, colour coded by $c - c^{\text{ref}}$. This should then look something like Fig. 7.

# 9 Conclusions

The proposed actions are:

1. Redefine the basis set $H_n(u)$ for $n > 0$ to be orthogonal to $H_0'(u)$ (see footnote 3). [Completed; see Jira C5CAL-1199]
2. Introduce the shift parameter $h_0$ in the CALIPD, using either of the formulations (1), (3), or (4). [Eq. (3) used; see NR slides[10]]
3. No specific change is needed to the LSF/PSF calibration, which is performed essentially as before, only with the added shift parameters and all their dependencies.
4. Implement the non-chromatic constraints as described in Sect. 5.1. [Done; see Jira C5CAL-1199]
5. Compute the functions $\bar{h}_{01}(t, w, g)$. This should be made available in the MDB in the form TBD.
6. Implement the constraints (8) in AGIS (TBD). As discussed in Sect. 5.2 this might be deferred to a later stage of Cycle 3 (TBC).
7. For the test, IDU needs an algorithm for modifying the source parameters by a given singular perturbation. The basic code is in Appendix B.
8. Select test data and implement the test described in Sect. 8 and Appendix A.

[10] wiki.cosmos.esa.int/gaia-dpac/index.php/File:2019.12.16_IDU_AGIS_NR.pdf

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# Acronyms

The following table has been generated from the on-line Gaia acronym list:

| Acronym | Description |
|---|---|
| AC | ACross scan (direction) |
| AGIS | Astrometric Global Iterative Solution |
| AL | ALong scan (direction) |
| BAM | Basic Angle Monitor |
| CCD | Charge-Coupled Device |
| CF | Conversion Factor [e- per ADU] |
| DR2 | Gaia Data Release 2 |
| ELSF | Empirical LSF (”option 4”) |
| ICRS | International Celestial Reference System |
| IDU | Intermediate Data Update |
| IPD | Image Parameter Determination |
| LSF | Line Spread Function |
| MDB | Main DataBase |
| NR | Nightly Run |
| NSL | Nominal Scanning Law |
| OBMT | On-Board Mission Timeline |
| PSF | Point Spread Function |
| SRS | Scanning Reference System |
| TBC | To Be Confirmed |
| TBD | To Be Defined (Determined) |
| TCB | Barycentric Coordinate Time |
| TDI | Time-Delayed Integration (CCD) |
| TN | Technical Note |

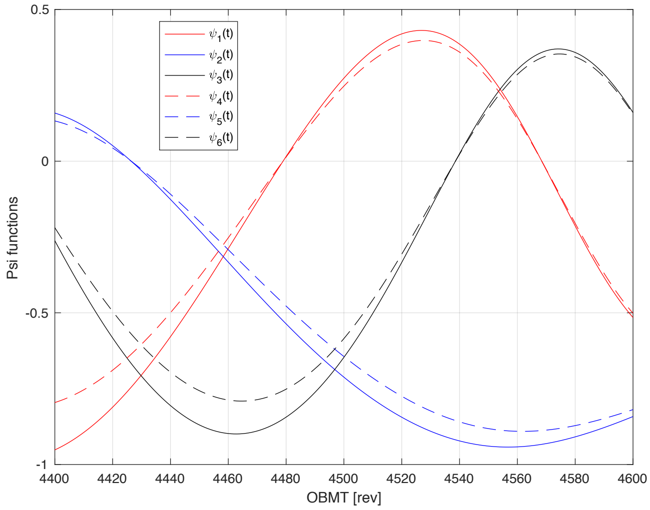


FIGURE 5: The six psi functions for the time interval OBMT 4400 to 4600 rev ($t_{\rm ref} = 2016.0$).

# Appendix A: Proposed test configuration

The test should use a selection of sources observed in a certain time interval, long enough for the psi functions to change significantly, i.e. a significant fraction of the NSL precession period of about 63 days (∼250 rev). A suitable interval could be OBMT $= 4400$–$4600$ rev (2016.836–2016.973), already used for a number of different tests.

Using $t^{\rm ref} = 2016.0$ as the reference epoch, the psi functions in this interval are as shown in Fig. 5. Recall that $\psi_k(t)$ for $k = 1 \dots 3$ are simply the (ICRS) $X$, $Y$, $Z$ components of the SRS $\boldsymbol{z}$ axis, and $\psi_{k+3}(t) = (t - t^{\rm ref})\psi_k(t)$.

The singular perturbation is defined by the 12 coefficients $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\omega}_i$ for $i = 0,\ 1$, where $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\omega}_0$ are the coefficients of $\psi_1(t) \dots \psi_6(t)$ for the non-chromatic perturbation, and $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\omega}_1$ are the coefficients of $\psi_1(t) \dots \psi_6(t)$ for the chromatic perturbation.

Rather than assigning random values to the 12 coefficients, it may be wise to give the perturbation some distinct and easily recognisable signatures in their non-chromatic and chromatic components.

Because the whole interval is approximately 0.8 to 1.0 year after the reference epoch, the psi functions are strongly correlated in the sense that $\psi_4$ to $\psi_6$ are roughly equal to 0.9 times $\psi_1$ to $\psi_3$. It is then in principle sufficient to perturb only the positions (non-zero $\boldsymbol{\varepsilon}_i$), or only the proper motions (non-zero $\boldsymbol{\omega}_i$). However, in order to exercise all parts of the perturbation code, the effect may be distributed equally between the positions and proper motions.

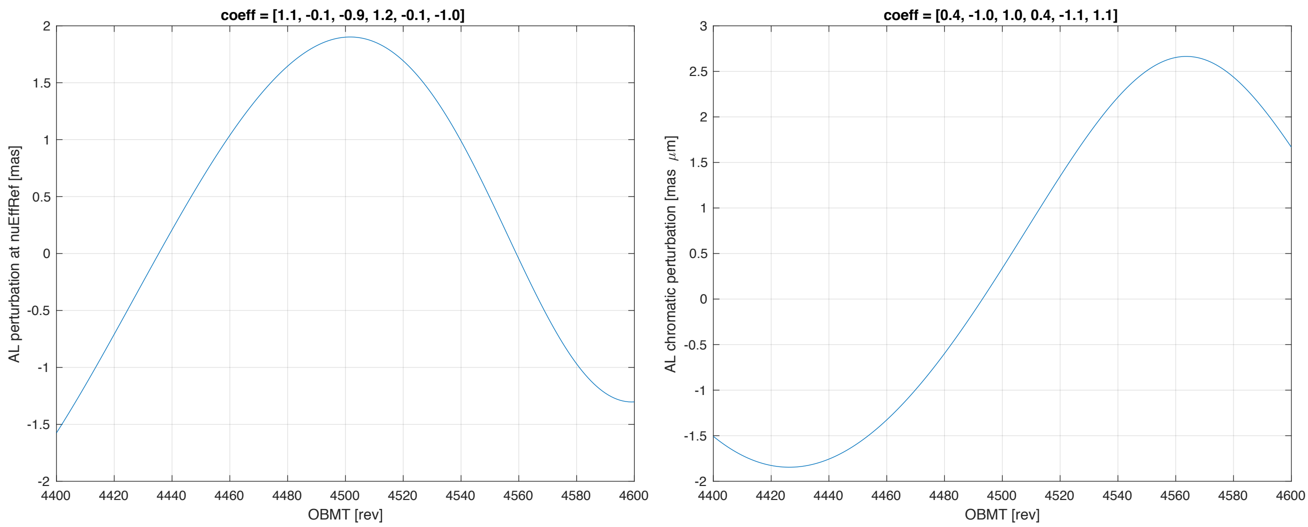


FIGURE 6: The non-chromatic (left) and chromatic (right) components of the singular perturbation.

The proposed values for the perturbation are as follows:

$$\varepsilon_0 = \begin{bmatrix} 1.1 \\ -0.1 \\ -0.9 \end{bmatrix} \text{ mas}, \quad \omega_0 = \begin{bmatrix} 1.2 \\ -0.1 \\ -1.0 \end{bmatrix} \text{ mas yr}^{-1}, \tag{12}$$

$$\varepsilon_1 = \begin{bmatrix} 0.4 \\ -1.0 \\ 1.0 \end{bmatrix} \text{ mas}, \quad \omega_1 = \begin{bmatrix} 0.4 \\ -1.1 \\ 1.1 \end{bmatrix} \text{ mas yr}^{-1}. \tag{13}$$

The corresponding AL effects (obtained as linear combinations of the psi functions) are shown in Fig. 6. The non-chromatic and chromatic components are nearly orthogonal.

To do the test, two complete runs of CALIPD+IPD would be executed on the same CCD data in OBMT 4400–4600, one using the unperturbed AGIS source parameters and one using the perturbed values. The perturbation of the source parameters could be done as sketched in Appendix B. The resulting two sets of AstroElementaries are compared by computing the differences

$$\Delta t_{\text{obs}} = t_{\text{obs}}^{\text{perturbed}} - t_{\text{obs}}^{\text{unperturbed}} \tag{14}$$

In a successful test the expected differences are

$$\Delta t_{\text{obs}} \simeq 0 \times p_0(t) - (c - c^{\text{ref}}) \times p_1(t), \tag{15}$$

where $p_0(t)$ and $p_1(t)$ are the functions shown in Fig. 6. The factor 0 in the first term of (15) comes from the application of the non-chromatic constraint in (7), while the second term is the full chromatic perturbation since no chromatic constraint is applied in the test. The negative sign is a consequence of $\partial t_{\text{obs}}/\partial\eta < 0$.

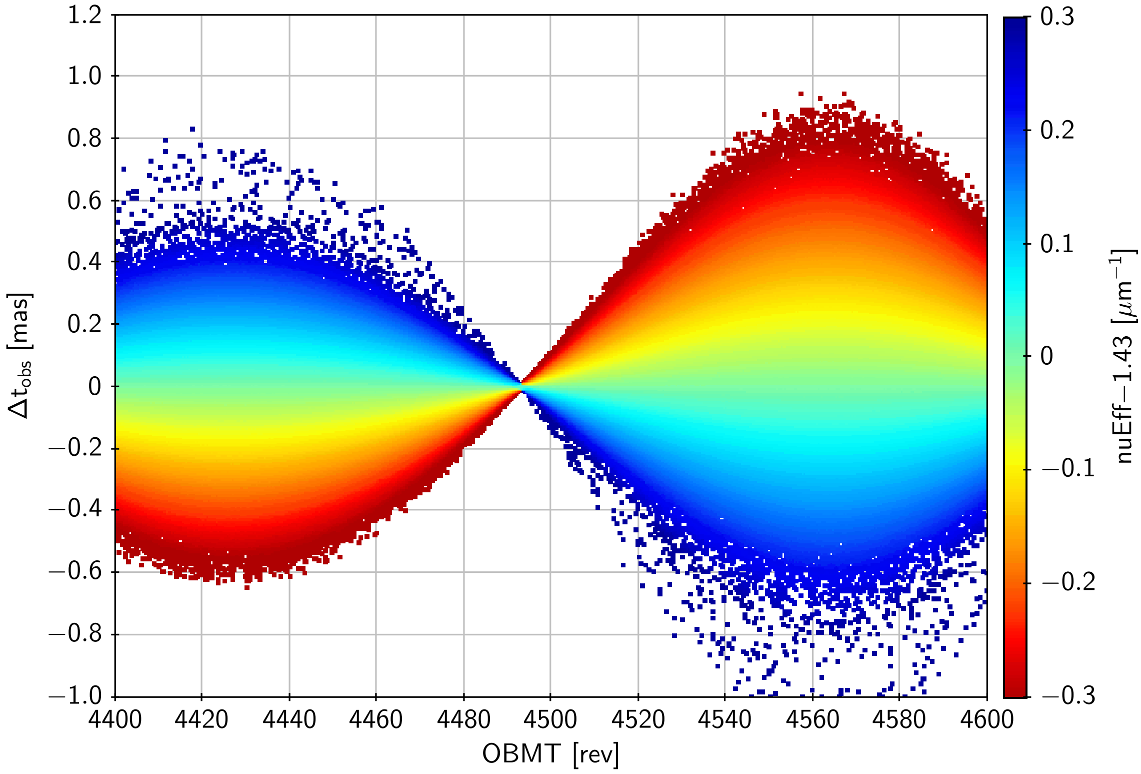


FIGURE 7: Expected general appearance of $\Delta t_{\rm obs}$ versus OBMT due to the chromatic perturbation.

Since $|p_1(t)| < 3$ mas $\mu$m, we expect

$$|\Delta t_{\rm obs}| < 0.03 \text{ mas} \tag{16}$$

for sources with $|c - c^{\rm ref}| < 0.01\ \mu\text{m}^{-1}$, i.e. where $\nu_{\rm eff} = 1.42$–$1.44\ \mu\text{m}^{-1}$. This concludes the non-chromatic test.

For the chromatic test it may be sufficient to show that $\Delta t_{\rm obs}$ versus OBMT has the expected general distribution shown in Fig. 7. An alternative (or rather complementary) test would be to compute

$$\Delta\bar{h}_{01}(t,w,g) = \bar{h}_{01}(t,w,g)^{\rm perturbed} - \bar{h}_{01}(t,w,g)^{\rm unperturbed}\,, \tag{17}$$

which should equal $p_1(t)$, the right digram in Fig. 6 (possibly with the reversed sign), for all combinations of $wg$.

# Appendix B: Perturbing the source parameters

The following code illustrates how the perturbation could be done, except that AstrometricSource does not have a method getWaveNumber (this should probably use DpcbSource instead), and the hardcoded values should be set somewhere else.

```
public void perturbSource(AstrometricSource src) {

// define perturbation
GVector3d epsilon0 = new GVector3d(1.1, -0.1, -0.9); // [mas]
GVector3d omega0 = new GVector3d(1.2, -0.1, -1.0); // [mas/yr]
GVector3d epsilon1 = new GVector3d(0.4, -1.0, 1.0); // [mas*micron]
GVector3d omega1 = new GVector3d(0.4, -1.1, 1.1); // [(mas/yr)*micron]

// reference value for nuEff in 1/micron:
double nuEffRef = 1.43;

// set upp transformation matrix R such that [da;dd]=R*epsilon
// (3rd row is not used)
double ca = Math.cos(src.getAlpha());
double sa = Math.sin(src.getAlpha());
double cd = Math.cos(src.getDelta());
double sd = Math.sin(src.getDelta());
GMatrix3d R = new GMatrix3d(ca*sd, sa*sd, -cd, -sa, ca, 0.0, 0.0, 0.0, 0.0);

// calculate epsilon and omega for the specific nuEff of the source
double deltaNuEff = src.getWaveNumber() * 1e3 - nuEffRef;
GVector3d epsilon = epsilon0.clone().scaleAdd(deltaNuEff, epsilon1);
GVector3d omega = omega0.clone().scaleAdd(deltaNuEff, omega1);

// calculate perturbation in position and pm (3rd element is not used)
GVector3d dPos = R.timesNew(epsilon); // [mas]
GVector3d dPm = R.timesNew(omega); // [mas/yr]

// apply perturbations
src.setAlpha(src.getAlpha() + GMath.masToRad(dPos.get(0)) / cd);
src.setDelta(src.getDelta() + GMath.masToRad(dPos.get(1)));
src.setMuAlphaStar(src.getMuAlphaStar() + dPm.get(0));
src.setMuDelta(src.getMuDelta() + dPm.get(1));

}
```