OGA1 Process Description

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Abstract

This document explains the theoretical basis for the OGA1 determination, i.e. the complete description of the EKF model implemented for the attitude improvement.

Document History

Issue	Revision	Date	Author	Comment
4	0	2009-10-12	DMP	version 4 issued
D	3	2009-10-07	BAS	Addition of Section 6 (which transits); some typos and clarifications
D	2	2009-08-27	DMP	Some general revisions from Bastian.
D	1	2009-08-24	DMP	Update the DMP-001-03 version using DPAC template. General review.

Contents

1 Introduction

The principal feature of the Gaia astrometry mission is a highly stable payload consisting of two scientific instrument telescopes with one big focal plane containing an array of many CCDs. In order to achieve these high-stability requirements the Gaia spacecraft should have accurate *on-board* attitude determination sensors and also accurate control actuators. On the other hand, the on-ground processed data should utilize more accurate attitude determination for further scientific ground processing of the Gaia observations.

The main objective of the task described in the present document is to reconstruct the *non-real-time* First On-Ground Attitude (OGA1) for the Gaia mission with very high accuracy for further processing. The accuracy requirements for the OGA1 determination (along and across scan) can be set to 50 milliarcsec for the first 9 months, to be improved later on in the mission to 5 milliarcsec.

After designing the final structure of the OGA1 algorithm, this will be implemented by writing the final version in JAVA and then integrated in the IDT chain.

2 Gaia System Model and Measurements

The model is intended to be “as simple as possible”. In other words, the satellite is represented by an *inertially rotating rigid body.* In fact it will follow the Nominal Scanning Law (NSL), which prescribes a constant scan rate of 60 arcsec per second ($3 \cdot 10^{-4}\, rad/s$) and a slow variation of the scanning plane of Gaia (precession of the scan axis). The angular acceleration of the satellite due to the precession will be approximately equal to $3 \cdot 10^{-10}\, rad/s^2$.

The state vector will consist of 7 elements: the attitude quaternion and the S/C angular velocity.

The measurement rate for 10^6 attitude stars on the whole sky (this is a very conservative assumption) is about 0.6 star per second. More precisely, it is about 0.3 star per second per field of view.

3 The Extended Kalman Filter model

3.1 System Model

The state vector x for the Gaia OGA1 KF combines estimation of orientation ($\mathbf{q}$) and angular velocity (ω) of the S/C w.r.t. the Satellite Reference System (SRS), see BAS-003.

$$x = \begin{pmatrix} \mathbf{q} \\ \omega \end{pmatrix} \tag{1}$$

It has seven components, four coming from attitude quaternion representation $\mathbf{q} = \begin{pmatrix} q_x & q_y & q_z & q_w \end{pmatrix}$ and three from the spin rate vector $\omega = \begin{pmatrix} \omega_x & \omega_y & \omega_z \end{pmatrix}$.

The system model is fully described by two sets of differential equations, the first one describing the satellite's attitude following the quaternion representation

$$\dot{\mathbf{q}}(t) = \frac{1}{2}\Omega(\omega)\mathbf{q}(t) \tag{2}$$

where

$$\Omega(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{3}$$

and the second one using the Euler's equations

$$\dot{\omega}(t) = I_{sc}^{-1}\left(T_e - \omega \times I_{sc}\omega\right) \tag{4}$$

where I_{sc} is the moment of inertia of the satellite and T_e is the total disturbance and control torques acting on the S/C.

As first assumption, the satellite is represented as a *freely rotating rigid body*, which implies to set the external torques to zero in the Eq.(4). If this simplification will not work nicely to reconstruct the Gaia's attitude, then the proper T_e required to follow the NSL should be taken into account.

3.2 Process and Measurement Model

The process model predicts the evolution of the state vector x and describes the influence of a random variable $\mu(t)$, the process noise. For non-linear systems, the process dynamics is described as following:

$$\dot{x}(t) = f(x(t), t) + G(x(t), t)\,\mu(t) \tag{5}$$

where f and G are functions defining the system properties. For OGA1, f is given by Eqs. 2 and 4, and $\mu(t)$ is a discrete Gaussian white noise process with variance matrix $Q(t)$:

$$\mu(t) \sim M(0, Q(t)) \tag{6}$$

The measurement model relates the measurement value y to the value of the state vector x and describes also the influence of a random variable $\nu(t)$, the measurement noise of the measured value. The generalized form of the model equation is:

$$y_k = h(x(t_k), t) + \nu(t) \tag{7}$$

where h is the function defining the measurement principle, and $\nu(t)$ is a discrete Gaussian white noise process with variance matrix $R(t)$:

$$\nu(t) \sim N(0, R(t)) \tag{8}$$

In order to estimate the state, the equations expressing the two models must be linearized in order to use the KF model equations, around the current estimation (x_k^-) for propagation periods and update events.

This procedure yields the following two matrices to be the Jacobian of f and h functions w.r.t. the state.

$$F = \left.\frac{\partial f(x(t), t)}{\partial x(t)}\right|_{x=x_k^-} \tag{9}$$

$$H_k = \left. \frac{\partial h\left(x\left(t_k\right), t\right)}{\partial x\left(t\right)} \right|_{x=x_k^-} \tag{10}$$

3.3 KF propagation equations

The KF propagation equations consist of two parts: the state system model and the state covariance equations. The first one

$$\dot{x}(t) = F(t)x(t) + G(t)\mu(t) \tag{11}$$

can be propagated using a numerical integrator, such as the the fourth-order Runge-Kutta method. The F matrix is called the transition matrix, Q the system noise covariance matrix and G the system noise covariance coupling matrix.

The transition matrix can be expressed as derived from MS-001:

$$F = \begin{bmatrix} 0.5\Omega(\omega) & 0.5\Xi(\mathbf{q}) \\ 0_{3\times4} & F_{\dot{\omega}\omega} \end{bmatrix} \tag{12}$$

where

$$\Xi(\mathbf{q}) = \begin{bmatrix} q_w & -q_z & q_y \\ q_z & q_w & -q_x \\ -q_y & q_x & q_w \\ -q_x & -q_y & -q_z \end{bmatrix} \tag{13}$$

$$F_{\dot{\omega}\omega} = -(I_{sc}^{-1}\left(\left[\omega\times\right] I_{sc} - \left[\left(I_{sc}\omega\right)\times\right]\right)) \tag{14}$$

Here the matrix notation $[a\times]$ represents the skew symmetric matrix of the generic vector a.

For the state covariance propagation, the Riccati formulation is used:

$$\dot{P} = FP + PF^T + GQG \tag{15}$$

Its prediction can be carried out through the application of the fundamental matrix Φ (i.e. first order approximation using the Taylor series) about F which becomes now.

$$\Phi(\Delta t) \approx I + F \cdot \Delta t \tag{16}$$

where Δt represents the propagation step.

The process noise matrix Q used for the Riccati propagation equation (15) is considered to be

$$GQG^T = diag\left(\left[\left(10^{-8}\right)^2, \left(10^{-8}\right)^2, \left(10^{-8}\right)^2, \left(10^{-8}\right)^2, \left(10^{-8}\right)^2, \left(10^{-8}\right)^2, \left(10^{-8}\right)^2\right]\right) \tag{17}$$

since OGA1 will depend more on the measurements (even if not so accurate at this stage) than on the system dynamic model. Nevertheless, in order to get an optimal accuracy of the estimation, a further tuning for the matrix will be necessary [ref. Padeletti (2009)].

3.4 KF update equations

The KF update equations correct the state and the covariance estimates with the measurements coming from the satellite. In fact, the measurement vector y_k consists of the so called *measured along scan angle* η_m, and the *measured across scan angle* ζ_m, and they are the values as read from the AF1 CCD.

On the other hand, the so called *calculated field angles* $(h(x_t) = [\eta_c, \zeta_c])$ are the FAs calculated from an ASC for each time of observation.
The set of the update equations are down listed.

$$\begin{cases} \hat{x}_k^+ & = \hat{x}_k^- + K_k \left[y_k - h_k \left(\hat{x}_k^- \right) \right] \\ P_k^+ & = \left[I - K_k H_k \left(\hat{x}_k^- \right) \right] P_k^- \\ K_k & = P_k^- H_k^T \left[H_k P_k^- H_k^T + R_k \right]^{-1} \end{cases} \tag{18}$$

where the measurement sensitivity matrix is given by

$$H_k = \begin{bmatrix} \frac{\partial \eta_k}{\partial \mathbf{q}} & 0_{1\times 3} \\ \frac{\partial \zeta_k}{\partial \mathbf{q}} & 0_{1\times 3} \end{bmatrix} \tag{19}$$

and the measurement noise matrix R is chosen such that

$$R = \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\zeta^2 \end{bmatrix} \tag{20}$$

The standard deviation for the FA errors along and across scan are computed and provided by IDT. The whole set of equations are summarized in following table.

		Definition Matrices
KF Dynamic Model	$\begin{aligned}\dot{\mathbf{q}}(t) &= \tfrac{1}{2}\Omega(\omega)\mathbf{q}(t)\\ \dot{\omega}(t) &= I_{sc}^{-1}\left(T_e - \omega \times I_{sc}\omega\right)\end{aligned}$	$\Omega(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}$
Error Covariance Propagation	$\dot{P} = FP + PF^T + GQG^T$	$GQG^T = diag\left[\ (10^{-8})^2 \quad (10^{-8})^2 \quad (10^{-8})^2 \quad (10^{-8})^2 \quad (10^{-8})^2 \quad (10^{-8})^2 \quad (10^{-8})^2 \quad (10^{-8})^2\ \right]$
	$\Phi \approx I + F \cdot \Delta t$	$F = \begin{bmatrix} 0.5\Omega(\omega) & 0.5\Xi(\mathbf{q}) \\ 0_{3\times4} & F_{\dot{\omega}\omega} \end{bmatrix}; \ \Xi(\mathbf{q}) = \begin{bmatrix} q_w & -q_z & q_y \\ q_z & q_w & -q_x \\ -q_y & q_x & q_w \\ -q_x & -q_y & -q_z \end{bmatrix}; \ F_{\dot{\omega}\omega} = -(I_{sc}^{-1}\left([\omega\times]\, I_{sc} - [(I_{sc}\omega)\times]\right))$
State Estimate Update	$\hat{x}_k^+ = \hat{x}_k^- + K_k\left[y_k - h_k\left(\hat{x}_k^-\right)\right]$	
Error Covariance Update	$P_k^+ = \left[I - K_k H_k\left(\hat{x}_k^-\right)\right]P_k^-$	$H_k = \begin{bmatrix} \frac{\partial \eta_k}{\partial \mathbf{q}} & 0_{1\times3} \\ \frac{\partial \zeta_k}{\partial \mathbf{q}} & 0_{1\times3} \end{bmatrix}$
Gain Matrix	$K_k = P_k^- H_k^T\left[H_k P_k^- H_k^T + R_k\right]^{-1}$	$R_k = \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\zeta^2 \end{bmatrix}$

TABLE 1: Summary of the equations used in the EKF.

4 OGA1 Processing Scheme

The OGA1 process scheme can be divided in 3 main parts: inputs, processing steps and outputs.

4.1 The Inputs

The input data sets are:

- Oga1Elementaries (OGA1 needs these in time sequence from IDT) composed essentially by:
  - transit identifiers (TransitId)
  - observation time (TObs)
  - observed FAs including geometry calibration
- A raw attitude (IOGA), with about 7 arcsec noise (in B-Splines).
- A cross-match table with pairs of: SourceId-TransitId, plus proper direction to the star at the instance of observation (calculated by IDT from an attitude star catalogue (ASC), with aberration, proper motion, parallax taken into account).

See Section6 for a more specific discussion about the question precisely *which* Oga1Elementaries, and *which* measurements within each Oga1Elementary, are to be used for OGA1.

4.2 The Processing Steps

The OGA1 determination is a KF process, i.e. essentially a loop over the individual observations, plus at the end a spline-fitting of the resulting quaternions. The main steps are:

1. Sort by time the list of Oga1Elementaries. Then, sort the list of cross-match sources by the transit identifier with the ones from the sorted list of Oga1Elementary. N.B.: the unmatched elementary transits are simply discarded.
2. Initialize KF: interpolate the B-Spline (IOGA attitude format) in order to get the first quaternion and angular velocity to start the filter. Optionally, the external torque can be reconstructed in order to have a better accuracy for the dynamical system model.
3. Forward KF: for a generic time t_i, we predict the attitude quaternion from the state vector at the time t_{i-1} of the preceding observation — or, at the very start of the filter process (step 'zero'), from the interpolated IOGA quaternion — using the system model described by the differential equations (2) and (4).

4. From the pixel coordinates we can compute the field coordinates from SM and AF measurements, using a Gaia calibration file. As observed field angles OGA1 considers the AF1 values.

5. The calculated field coordinates of known stars (from ASC) are so computed:

   (a) recovering star parameters from catalog, using cross-match tables;

   (b) computing apparent star position for the instant of observation, using star parameters, Gaia ephemeris and solar-system ephemeris;

   (c) applying predicted attitude quaternion for t_i: multiply apparent star position to get field coordinates.

6. Correct the state using the difference between the observed and the calculated measurements in the along-scan ($\Delta\eta$) and across-scan ($\Delta\zeta$) directions.

7. At the end of the loop over the measurements (i.e. at the end of the relevant time interval): generate then a B-Spline representation from the OGA1 quaternions $\overrightarrow{q}_{OGA1}(t_i)$ for the whole time interval.

8. The final step consists to hand the results back to IDT.

Differently from what stated in the previous drafts, the OGA1 determination process does not need to backward the KF anymore. Indeed, the high precision of the measurements provided by IDT (along with the low weight given to the simple system model) leads the KF process to consider big values for the measurement noise matrix R, which takes the filter to reach the accuracy requirements from the first simulation instants.

4.3 The Outputs

The improved attitude (for each observation time t_i) in two formats:

- quaternions $\overrightarrow{q}_{OGA1}(t_i)$ in the form of array of doubles.
- B-Spline representation from the previous OGA1 quaternions.

5 Process Diagram

The following diagram (Fig.1) gives just a schematic representation for the OGA1 process.

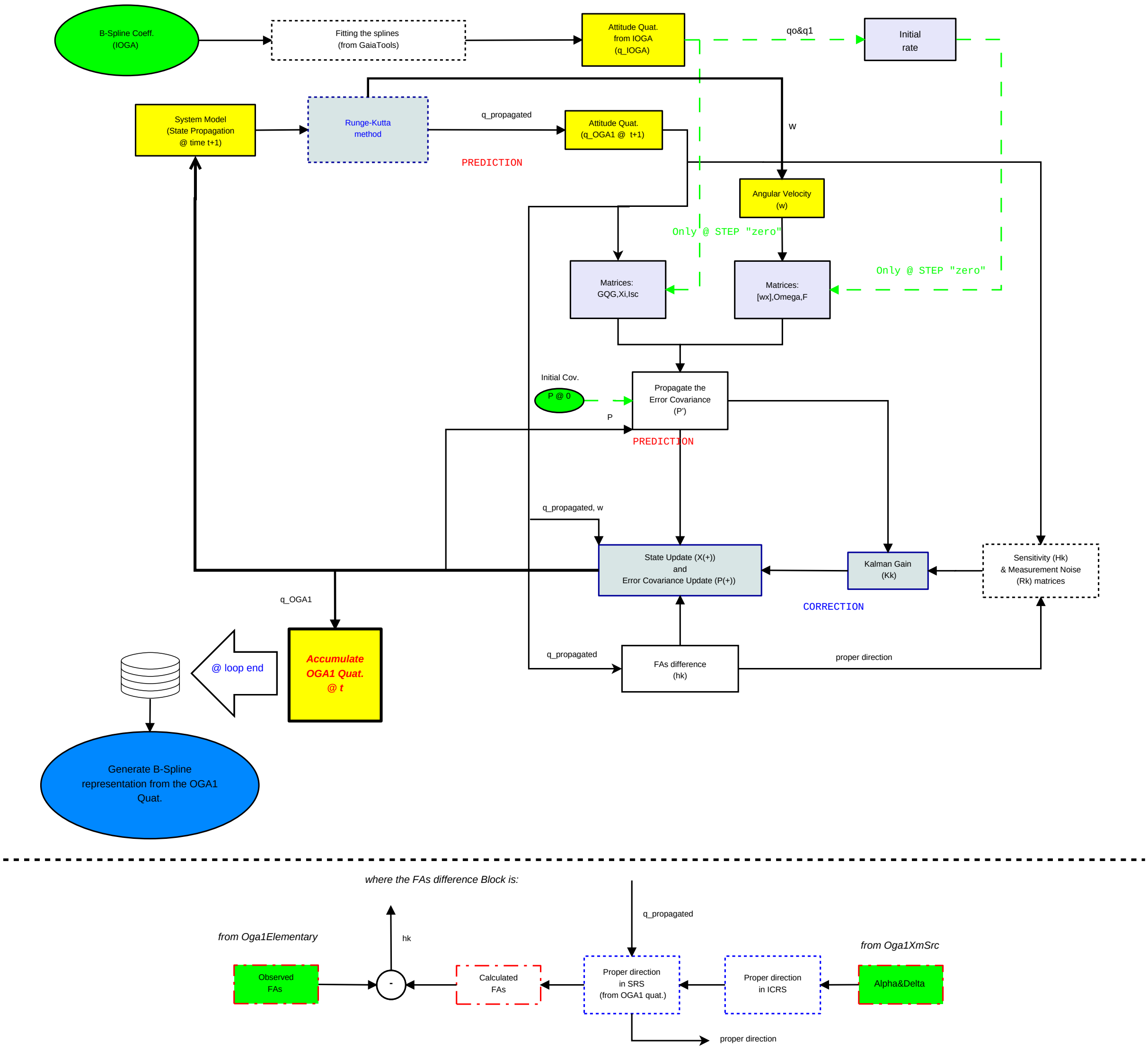

FIGURE 1: Actual process diagram for OGA1 determination

The difference block in the correction part leads to the difference between the measured and calculated FAs. The process steps are the following:

1. starting from the α and δ delivered by IDT in the Oga1 XM record, as described in BAS-003:

$$\mathbf{u}_{IOGA}^{ICRS} = \begin{cases} u_x = & \cos\theta\cos\alpha \\ u_y = & \cos\theta\sin\alpha \\ u_z = & \sin\theta \end{cases} \tag{21}$$

2. multiply now the proper direction with the OGA1 system model quaternion (for a given t_{obs}) to get the proper direction (SRS) using the OGA1 system model

$$\mathbf{u}_{OGA1}^{SRS} = A\left(\mathbf{q}_{OGA1}\right)\mathbf{u}_{IOGA}^{ICRS} \tag{22}$$

where the attitude transformation matrix A is

$$A\left(\mathbf{q}_{OGA1}\right) = \begin{bmatrix} q_x^2 - q_y^2 - q_z^2 + q_w^2 & 2\left(q_x q_y + q_z q_w\right) & 2\left(q_x q_z - q_y q_w\right) \\ 2\left(q_x q_y - q_z q_w\right) & -q_x^2 + q_y^2 - q_z^2 + q_w^2 & 2\left(q_y q_z + q_x q_w\right) \\ 2\left(q_x q_z + q_y q_w\right) & 2\left(q_y q_z - q_x q_w\right) & -q_x^2 - q_y^2 + q_z^2 + q_w^2 \end{bmatrix} \tag{23}$$

3. calculate the FAs by this proper direction vector, using the inverse of Eq. (24)

$$\mathbf{u}_{OGA1}^{SRS} = \begin{cases} u_x = & \cos\zeta^c\cos\left(\eta^c \pm \frac{\gamma}{2}\right) \\ u_y = & \cos\zeta^c\sin\left(\eta^c \pm \frac{\gamma}{2}\right) \\ u_z = & \sin\zeta^c \end{cases} \tag{24}$$

where the plus sign is for the preceding FoV ($i = 1$), and the minus sign for the following FoV ($i = 2$).

6 Which stars and which measurements?

6.1 Which stars: Composition of the attitude star catalogue

OGA1 must use two-dimensional (2-d) astrometric measurements from Gaia. That is, the CCD transits of stars used by OGA1 must have produced 2-d windows. OGA1 furthermore requires at least one such 2-d measurement per second and per FoV, in order to obtain the required precision. This corresponds to a minimum star density of 75 per square degree.

The easiest choice for the OGA1 transits and the attitude star catalogue members would be the ”bright” stars, G<13, for which 2-d windows are always produced. However, as demonstrated in M. Biermann (2009), those stars add up to only 50 per square degree in the polar caps of the Milky Way. Therefore M. Biermann (2009) proposes to add more bright transits with 2-d windows by defining some on-board detections with G>13 as calibration faint stars (CFS).

Issue 1 of M. Biermann (2009) proposes two possible solutions: either all transits with $13.0 < G < 13.4$, or 35 percent of all transits with $13.0 < G < 14.0$. Both solutions are estimated to provide the needed rate of OGA1 transits even at the galactic poles. The final decision will be taken by the Calibration Working Group very soon.

Depending on that decision, the attitude star catalogue would thus have to consist of all stars brighter than G=13.6 (perhaps with some density reduction along the Milky Way to keep the catalogue small, and thus to keep both the IDT/OGA1 cross-matching and the OGA1 process itself computationally small), or of all stars brighter than G=14.2 (again perhaps with some density reduction along the Milky Way). A margin of 0.2 magnitudes has been adopted here to care for the errors of the on-board magnitude estimation from the SM transits.

The necessary positional information for the creation of the attitude star catalogue can be taken from the UCAC-3 catalogue of the US Naval Observatory for the majority, and from the Hipparcos and Tycho Catalogues for the brighter stars. The density reduction along the Milky Way is not mandatory, but should be done if by all means possible. If it is done, it could also try to select well-isolated stars.

The main requirement on the attitude star catalogue, however, remains the minimum density of 75 stars observed in 2-d mode per square degree, on each and every Gaia field of view (which spans about half a square degree), in other words: about 20 stars observed in 2-d mode on every 0.25 square degrees field on the sky.

This density must be achieved within the magnitude range(s) and CFS percentages given above, for the operational reasons discussed above. In the first case (100 percent to G<13.4), it means a total star density of the attitude star catalogue to G=13.6 of about 90 per square degree, in the second case (35 percent for $13.0 < G < 14.0$) it means a total star density of the attitude star catalogueto G=14.2 of about 120 per square degree, [1] composed of about 50 for G<13 and 70 for $13.0 < G < 14.2$.

[1] Up to G=13 the sky gives about 50 per square degree, so 25 are needed in addition. Due to the 35-percent CFS selection rate, an observation rate of 25 belongs to a star density of 70 per square degree

6.2 Which measurements per transit?

The OGA1 process, being a Kalman filter, needs its measurements in strict time sequence. In order to keep the OGA1 process simple, the baseline is thus to use only one CCD transit per Oga1Elementary, i.e. per field-of-view transit of an attitude catalogue star. Using more than one CCD transit per Oga1Elementary would imply a complex administration of the Oga1Elementaries, the CCD transits of which overlap in time due to the long duration of each field-of-view transit.

Having said this, it is the AF1 transit that naturally lends itself to be used. In the very first OGA1 processing step (Section 4.2), the Oga1Elementaries are sorted by time, which is naturally the AF1 transit time embedded in the transit identifier.

What if the attitude star catalogue cannot be made dense enough, or if the 2-d windows cannot be made numerous enough to guarantee a sufficiently dense set of Oga1Elementaries?

In that emergency case it would indeed be necessary to use more than one CCD transit per Oga1Elementary. An easy way to rescue the simple OGA1 processing logic (i.e. to save oneself from developing a complex administration of the Oga1Elementaries) would be to simply duplicate all elementaries, label the copies with the transit time on (say) AF7 instead of AF1 (plus a flag saying it is a copy), and then re-sort (merge) the duplicate set with the original one. This would allow to use transits from AF1 and AF7, i.e. two CCD transits per Oga1Elementary. The only addition to the processing logic is to check the copy flag. In a similar manner, three or four CCD transits could be used by creating (and labelling) more copies of the Oga1Elementaries.

Acronyms

The following table has been generated from the on-line Gaia acronym list:

Acronym	Description
AF	Astrometric Field (in Astro)
ASC	Attitude Star Catalogue
CCD	Charge-Coupled Device
DPAC	Data Processing and Analysis Consortium
EKF	Extended Kalman Filter
FA	Field Angle
FoV	Field of View (also denoted FOV)
Gaia	Global Astrometric Interferometer for Astrophysics
ICRS	International Celestial Reference System
IDT	Initial Data Treatment
IOGA	Initial On-Ground Attitude
KF	Kalman Filter
NSL	Nominal Scanning Law
OGA1	First On-Ground Attitude determination (in IDT)
SM	Sky Mapper
SRS	Scanning Reference System
S/C	Spacecraft
XM	Cross-Matching

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