## An introduction to the NIRSpec parametric model


#### Abstract

: This note provides a brief overview of the NIRSpec parametric model and a help to navigate existing documentation.


## 1 INTRODUCTION

NIRSpec (Near Infrared Spectrograph) is the near infrared spectrograph of the James Webb Space Telescope (JWST). The main elements and basic characteristics of the instrument are listed in Table 1 (reproduced here from Ref. [1]).
For simulation purposes and to asses in detail the instrument performances, it is necessary to be able to compute the path followed by the light entering NIRSpec. The NIRSpec parametric model was originally developed to make predictive "forward" simulations of NIRSpec data and it provides the basis of the NIRSpec Instrument Performance Simulator (IPS). Naturally, this same model, once properly tuned to reflect the calibration data acquired during NIRSpec cryo-vacuum test campaigns, provides also the "backward" transformations to understand the nature of light falling onto the detectors, and as such, it can be used to i) enable the extraction of NIRSpec spectra and to ii) aid the required computations for the onboard target acquisition procedure.
This note provides a brief introduction to the NIRSpec parametric model. The note is part of a package of documents containing detailed descriptions of the model parameters, definitions and file formats, which is being delivered by the ESA Science Operation Team to STScI. All these documents are listed in Table 2. Note that these documents were originally written to document the interface and methods of the IPS and this is often mentioned in their text. For the purpose of understanding the documents, one has to bear in mind that the NIRSpec parametric model is a fundamental element of the IPS and the model interface and methods are the same whether one has to use it for simulation purposes or spectral trace computations (since both operations require the computation of light-paths).
In the following section, we provide an overview of the NIRSpec parametric model and a starting point to navigate the reference documents.

Table 1: NIRSpec basic characteristics.

| Feature | Value and explanation |
| :---: | :---: |
| Spectral range | $0.6 \mu \mathrm{~m}-5.0 \mu \mathrm{~m}$ |
| Field of view | $>9 \operatorname{arcmin}^{2}$ |
| Spectral resolutions | $\begin{aligned} & R \approx 100 \text { (low) } \\ & R \approx 1000 \text { (medium) } \\ & R \approx 2700 \text { (high) } \end{aligned}$ |
| Spectral bands (gratings) | Band I ( $1.0 \mu \mathrm{~m}-1.8 \mu \mathrm{~m}$ ) <br> Band II ( $1.7 \mu \mathrm{~m}-3.0 \mu \mathrm{~m}$ ) <br> Band III ( $2.9 \mu \mathrm{~m}-5.0 \mu \mathrm{~m}$ ) <br> Band $0.7(0.7 \mu \mathrm{~m}-1.2 \mu \mathrm{~m})$ |
| Filters | CLEAR (complete range) <br> F100LP ( $1.0 \mu \mathrm{~m}-5.0 \mu \mathrm{~m}$, long-pass) <br> F170LP ( $1.7 \mu \mathrm{~m}-5.0 \mu \mathrm{~m}$, long-pass) <br> F290LP ( $2.9 \mu \mathrm{~m}-5.0 \mu \mathrm{~m}$, long-pass) <br> F070LP ( $0.7 \mu \mathrm{~m}-5.0 \mu \mathrm{~m}$, long-pass) <br> F110W ( $0.99 \mu \mathrm{~m}-1.3 \mu \mathrm{~m}$, target acquisition band-pass) <br> F140X ( $0.8 \mu \mathrm{~m}-2 \mu \mathrm{~m}$, target acquisition band-pass) <br> OPAQUE (closed, internal calibration and pupil reference) |
| MSA elements | Four quadrants of $365 \times 171$ microshutters, width 200 mas |
| Fixed slits | SLIT_A_200_1, SLIT_A 200_2, SLIT_B_200: width 200 mas SLIT_A_400: width 400 mas SLIT_A_1600: square aperture, width 1600 mas |
| IFU | 30 slices, width 100 mas, length 3 arcsec each |
| GWA elements | PRISM ( $R \approx 100$ ) <br> G140M, G235M, G395M: gratings $R \approx 1000$, bands I-III G140H, G235H, G395H: gratings $R \approx 2700$, bands I-III TAM: Target Acquisition Mirror |
| Detectors | 2 Mercury-Cadmium-Telluride Sensor Chip Arrays (SCAs), $5.3 \mu \mathrm{~m}$-cut off, each $2048 \times 2048$ pixels of $18 \mu \mathrm{~m} \times 18 \mu \mathrm{~m}$ ( $100 \times 100 \mathrm{mas}^{2}$ ), labeled 491 (blue side) and 492 (red side) |

mas $=$ milliarcsec

## 2 THE NIRSPEC PARAMETRIC MODEL



Figure 1: Paraxial layout of the NIRSpec optical train with the main modules and nominal focal ratios. Pupils are indicated in red, focal planes in blue, the optical modules are indicated by black arrows. The system field stop is located at the OTEIP. The IFU optics is coupled into the beam at the MSA focal plane (Ref. [1])

As schematically shown in Fig. 1, the optical design of NIRSpec consists of three major blocks and the light-path within the instrument can be described as follows. The light entering NIRSpec from the telescope image plane (OTEIP = optical telescope element image plane), is re-imaged by the FORE optics onto the slit plane of the Micro-Shutter Array (MSA). In the pupil plane of the FORE optics, the spectral band is selected by one of the filters in the Filter Wheel Assembly (FWA), as specified in Table 1. From the MSA slits, the light then is projected by the COLlimator optics (COL), onto the Grating Wheel Assembly (GWA), where a second pupil plane is located. Here, the light is dispersed by one of the seven disperser elements of the GWA described in Table 1 or reflected by the mirror for imaging. Finally, the CAMera optics (CAM) focuses the (dispersed) beam onto the two detectors in the Focal Plane Array (FPA) (see also Ref [1]).

Table 2: Reference documents for the NIRSpec parametric model.

| Ref. | Title | Author | Document number |
| :---: | :--- | :--- | :---: |
| 1 | Verification and science simulations with the <br> IPS for JWST/NIRSpec | B. Dorner | Ph. D. Thesis |
| 2 | NIRSpec parametric model ICD | G.Giardino \& IPS <br> development team | NTN-2013-013 |
| 3 | Parameterization of the coordinate transforms <br> in NIRSpec | P. Ferruit \& IPS de- <br> velopment team | NIRS-CRAL-MO1004 |
| 4 | NIRSpec coordinate transforms in MOS and <br> SLIT modes | P. Ferruit | NIRS-CRAL-TN-1080 |

The Integral Field Unit (IFU) entrance aperture is located in the MSA plane. IFU and MSA observations are mutually exclusive as their spectra share the same detector area, so all shutters have to be closed during IFU operations, and the IFU has to be blocked for multiobject exposures using the MSA. The IFU optics are split into an IFU FORE part, which reimages and scales the MSA plane onto the slicer, and an IFU POST part, which picks up the 30 image parts, and creates a virtual slit image for each slice at the MSA plane. The rest of the light path is similar to the other observation modes.
The NIRSpec parametric model encapsulate all the main optical elements identified in Fig. 1. There are two types of model components: the geometrical description of the key imageplane elements and the parameterization of the coordinate transforms between the planes.

### 2.1 Geometrical Parameters

The parameters used for the geometrical description of the model elements are listed in Table 3 together with the name of the files where these parameters are stored. Within the model directory structure all the description files listed in Table 3 are located in subdirectory Description. Each file content and format is described in detail in Ref. [2].

Table 3: Physical parameters used within the model for the geometrical description of the main optical elements of NIRSpec. For a detail description of the parameters and file format see Ref. [2].

| Element | Characteristic | Model parameters | File name |
| :--- | :--- | :--- | :--- |
| MSA | Fixed slit posi- <br> tions | Center x, y, and size in x and y | MSA.msa |
| MSA | Microshutter <br> positions | Per quadrant: Reference x, y, rota- <br> tion; per shutter: center x, y and <br> size in x and y | MSA.msa |
| IFU | Slicer positions | Slicer reference position x, y and <br> rotation. For each slicer entrance: <br> center x, y and size in x and y | IFU.sgd |
| GWA | Disperser prop- <br> erty and align- <br> ment | Element tilt angles $\theta_{x}, \theta_{y}, \theta_{z}$ plus <br> as-built y-tilt $\Theta_{y}$ and grating groove <br> density | disperser_G140M.dis, <br> disperser_G140H.dis, etc. |
| GWA | Disperser prop- <br> erty and align- <br> ment | Element tilt angles $\theta_{x}, \theta_{y}, \theta_{z}$ plus as- <br> built y-tilt $\Theta_{y}$ and prism dispersion <br> law parameters (6) | disperser_PRISM.pri |
| GWA | Sensor reading <br> calibration | y-tilt angle vs sensor reading pa- <br> rameterization (2 coefficients) | disperser_G140M_TiltY.gtp, <br> disperser_G140H_TiltY.gtp, <br> etc. |
| FPA | Detector posi- <br> tions | For each SCA: Reference x, y, rota- <br> tion, pixel pitch in x and y | FPA.fpa |

### 2.2 Coordinate Transforms

The other type of model components are the coordinate transforms between NIRSpec principal planes. The coordinate transforms in the NIRSpec model use a paraxial transform between the principal planes, departures from the ideal paraxial system are treated as distortions and modeled using a 2D polynomial (see Ref. [1] and Ref. [3]).
The paraxial part of a forward coordinate transform is defined by the magnification factors along the output axes $\gamma_{x}$ and $\gamma_{y}$, the rotation angle of the coordinate system $\theta$, and the position of the rotation center in the input and output reference frame ( $\mathrm{x}_{0 i n}, \mathrm{y}_{0 i n}$ ) and ( $\mathrm{x}_{0 \text { out }}$, $\mathrm{y}_{0 \text { out }}$ ). Given a point with coordinate ( $\mathrm{x}_{\text {in }}, \mathrm{y}_{i n}$ ) in the input focal plane, Eq.4.1-1 and Eq. 4.1-2 in Ref. [3] provide the formulas to compute its corresponding coordinate ( $\mathrm{x}_{\text {out }}, \mathrm{y}_{\text {out }}$ ) in the output focal plane.
The paraxial description of the optical modules is based on the nominal entrance and exit focal lengths, so in a transform between two focal planes, as e.g. in the FORE transform, where the coordinates are the positions in the local reference frame in unit of length, the magnification factors $\gamma_{x}$ and $\gamma_{y}$ correspond to the ratio of the focal lengths:

$$
\begin{equation*}
\gamma_{x}=\frac{f_{\text {outx }}}{f_{\text {inx }}}, \quad \gamma_{y}=\frac{f_{\text {outy }}}{f_{\text {iny }}} \tag{1}
\end{equation*}
$$

Transforms between a focal plane and a pupil plane (such as the COL transform between the MSA and the GWA) are performed with exactly the same formulas. In this case:

$$
\begin{equation*}
\gamma_{x}=\frac{1}{f_{i n x}}, \quad \gamma_{y}=\frac{1}{f_{i n y}} \tag{2}
\end{equation*}
$$

and the output coordinates are unit-less cosines ( $v_{x}, v_{y}$ ). The light paths through the dispersive elements of the GWA or the mirror, however, is more easily computed using direction cosine (see Ref. [4]). The expression to convert from cosines to direction-cosines is:

$$
\left(\begin{array}{c}
\cos a  \tag{3}\\
\cos b \\
\cos c
\end{array}\right)=\left(\begin{array}{l}
v_{x} /|v| \\
v_{y} /|v| \\
1 /|v|
\end{array}\right)
$$

with $|v|=\sqrt{v_{x}^{2}+v_{y}^{2}+1}$. The inverse operation to go from direction-cosines to cosines is therefore:

$$
\begin{equation*}
\binom{v_{x}}{v_{y}}=\binom{\cos a / \cos c}{\cos b / \cos c} \tag{4}
\end{equation*}
$$

More details about the coordinate transform approach in the model can be found in Ref. [1] and Ref. [3]. Additionally, in Ref. [3] the equations for the backward coordinate transforms can be found.
As indicated in Fig. 1, the main coordinate transforms of the NIRSpec parametric model are the FORE-optics transforms, the COLlimator transform and the CAMera transform. The paraxial parameters and distortion coefficients for all of these are stored in files with extensions .pcf in the model subdirectory called CoordTransform. The format of all these files is the same and is described in Ref. [2].
The parameter files for the IFU-FORE and IFU-POST coordinate transforms (there are 30 of this last ones, one for each virtual slit) are in subdirectory IFU of CoordTransform. A more
detailed description of the IFU-specific transforms is being prepared in a separate document, the document is not part of this package as the extraction of IFU spectra is beyond the scope is this first delivery.

### 2.3 The effect of the filters and the dispersive elements

The transmissive filters in the FWA cause chromatism that is a dependency of the light path on the light wavelength. In the NIRSpec parametric model this is accounted for via the distortion coefficients of the coordinate transform. In the case of the FORE transforms these coefficients are first-order wavelength-dependent polynomials of the form:

$$
\begin{aligned}
a_{i, j}(\lambda) & =\alpha_{x i, j} \lambda+\beta_{x i, j} \\
b_{i, j}(\lambda) & =\alpha_{y}{ }_{i, j} \lambda+\beta_{y i, j}
\end{aligned}
$$

where the coefficients $a_{i, j}$ and $b_{i, j}$ are the distortion coefficients in the formalism of Eq. 4.1-2 in Ref. [3]. Since each filter bends the light in a slightly different way, there is, in the model, a FORE transform for each of the seven transmissive filters listed in Table 1.
Finally, when transforming the coordinates of a point in the MSA plain to those of the same point in the FPA plane one has to take into account the effect of the mirror or the dispersive elements on the light path in the GWA. This is done in the following steps:
i applying the COL forward coordinate transform to the point in the MSA plane to get the cosine coordinates of the point in the collimator pupil plane
ii from the cosine coordinates compute the direction-cosine vector using Eq. 3
iii compute the effect of the GWA element on the direction-cosine using the formulas specified in Ref. [4]
iv from the new direction-cosine vector compute the cosine coordinates using Eq. 4
v apply the forward CAM transform to get the FPA coordinate of the input point
The effect of the mirror on a vector at the exit of the collimator is given by Eq. 5.3.-1 of Ref. [4] and the effect of a grating or the prism are given, respectively, by Eq. 5.3.-2 and Eq. 5.3.-3 of Ref. [4]. In these formulas, the rotations of an input direction-cosine $\left(x_{i n}, y_{i n}, z_{i n}\right)$ around the $x, y$ and $z$ axis by the element tilt angles, $\theta_{x}, \theta_{y}, \theta_{z}$ and $\Theta_{y}$ are implemented in the following way and following order ${ }^{1}$.
Rotation around x-axis:

$$
\left(\begin{array}{l}
x_{1}  \tag{5}\\
y_{1} \\
z_{1}
\end{array}\right)=\left(\begin{array}{c}
x_{i n} \\
y_{i n} \cos \theta_{x}+z_{i n} \sin \theta_{x} \\
\sqrt{1-x_{1}^{2}-y_{1}^{2}}
\end{array}\right)
$$

[^0]Rotation around y-axis:

$$
\left(\begin{array}{l}
x_{2}  \tag{6}\\
y_{2} \\
z_{2}
\end{array}\right)=\left(\begin{array}{c}
x_{1} \cos \theta_{y}-z_{1} \sin \theta_{y} \\
y_{1} \\
\sqrt{1-x_{2}^{2}-y_{2}^{2}}
\end{array}\right)
$$

Rotation around z -axis:

$$
\left(\begin{array}{l}
x_{3}  \tag{7}\\
y_{3} \\
z_{3}
\end{array}\right)=\left(\begin{array}{c}
x_{2} \cos \theta_{z}+y_{2} \sin \theta_{z} \\
-x_{2} \sin \theta_{z}+y_{2} \cos \theta_{z} \\
\sqrt{1-x_{3}^{2}-y_{3}^{2}}
\end{array}\right)
$$

Rotation around y-axis by $\Theta_{y}$ :

$$
\left(\begin{array}{c}
x_{\text {out }}  \tag{8}\\
y_{\text {out }} \\
z_{\text {out }}
\end{array}\right)=\left(\begin{array}{c}
x_{3} \cos \Theta_{y}-z_{3} \sin \Theta_{y} \\
y_{3} \\
\sqrt{1-x_{\text {out }}^{2}-y_{\text {out }}^{2}}
\end{array}\right)
$$

The inverse rotations need then to be applied to a vector at the exit of the grating element before applying the CAM transform, in the exact reverse order with negative angles, so: first Eq. 8 with angle $-\Theta_{y}$, followed by Eq. 7 with angle $-\theta_{z}$, followed by equation Eq. 6 with angle $-\theta_{y}$, followed by equation Eq. 5 with angle $-\theta_{x}$.

Note that Eq. 5.3.-3 that provides the coordinate transforms to be applied to a light ray at the exit of the collimator and going through the prism are expressed in terms of the prismmedium refraction index $n$, which is a function of wavelength. The relation is given by the Sellmeier equation:

$$
\begin{equation*}
n^{2}=1+\frac{K_{1} \lambda^{2}}{\lambda^{2}-L_{1}}+\frac{K_{2} \lambda^{2}}{\lambda^{2}-L_{2}}+\frac{K_{3} \lambda^{2}}{\lambda^{2}-L_{3}} \tag{9}
\end{equation*}
$$

where the values of the six parameters for the reference temperature and pressure are stored in Description file disperser_PRISM.dis.


[^0]:    ${ }^{1}$ Note that the order of the rotations given here is different to that specified in the equations in Ref. [4], since here we apply the rotation around the $y$-axis by $\Theta_{y}$ as last (at the GWA element entrance, and as first at the element exit) while in Ref. [4] this rotation is applied together with the rotation around the y -axis of $\theta_{y}$. The correct order in the latest model formalism is the one specified here.

