Scientific Requirements for Basic Angle Stability and Monitoring

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ABSTRACT. The scientific requirements for the basic angle stability and the monitoring of basic angle variations are reviewed in light of the need for absolute parallax determination, which is a primary goal of Gaia. Requirements are proposed for the random and systematic basic angle variations, and for the measurement of these variations.

1 Introduction

The present requirement for the stability of the basic angle is 7 μas rms over 6 hours. This number was derived by assuming that the basic-angle fluctuations contribute as a random noise in the astrometric error budget. This is approximately true for most kinds of basic-angle fluctuations on a time scale shorter than the spin period. However, it was pointed out in GAIA–LL–052 that there is one specific mode of basic-angle variation that is perfectly correlated with the astrometric parameters and in fact cannot be distinguished from a global shift in the parallax zero point. A significant such shift is incompatible with the overall scientific requirements of Gaia (cf. Appendix). A more detailed specification was therefore proposed in GAIA–LL–052 (Rev. 1, 8 May 2004), in which a separate (and much stricter) requirement was put on the relevant mode of variation.

However, it appears difficult from an engineering viewpoint to guarantee that this particular requirement will be satisfied. An alternative route has therefore been explored, which should make it possible to relax the stability requirement to an acceptable level while fully satisfying the scientific requirements. The present document aims to describe the problem more precisely, outline the methods available for its solution, and propose a revised set of requirements based on a combination of these methods.

2 Physical origin of a parallax bias

Random variations of the basic angle produce along-scan errors in the field transits that generally propagate to the astrometric parameters in a similar way as most other noise sources, such as photon noise: a certain 'coefficient of improvement' (much less than unity) applies, thanks to the statistical averaging-out of errors from the many scans across the same object. However, this may not apply to errors that are not truly 'random'. In particular, any error whose size and direction are somehow related to the geometry of the satellite with respect to the Sun might systematically affect the determination of
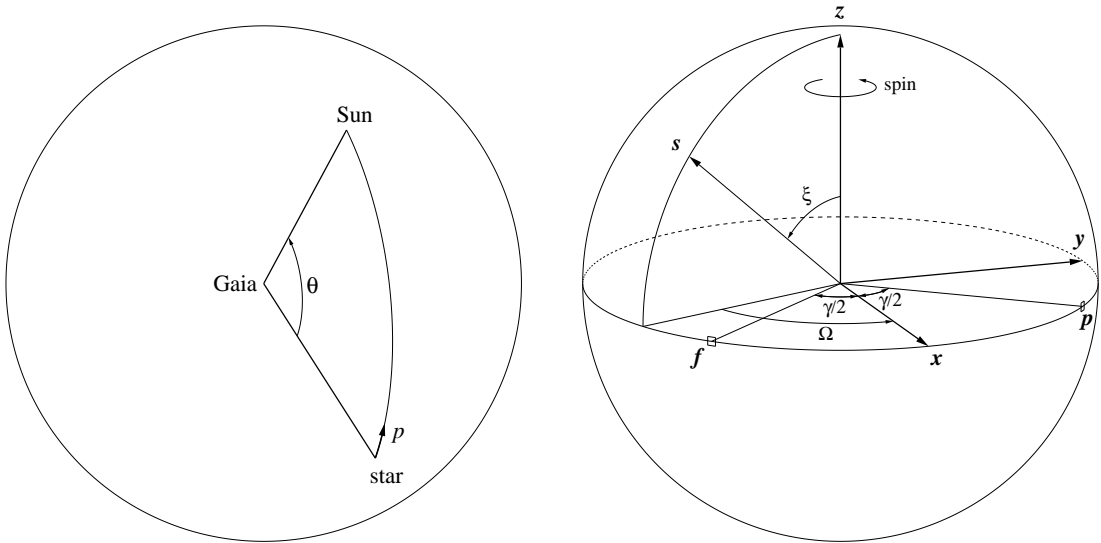
FIGURE 1: (Left) The parallactic displacement p of a star's celestial position is always along a great-circle arc from the star towards the Sun. The size of the displacement is given by Eq. (1) and depends on the angle θ between the star and the Sun.

FIGURE 2: (Right) Geometry of the Gaia Astro instrument ($\boldsymbol{xyz}$ axes and viewing directions $\boldsymbol{p}$, $\boldsymbol{f}$) with respect to the Sun ($\boldsymbol{s}$), defining the angles ξ (solar aspect angle), γ (basic angle), and Ω (heliotropic spin phase).

parallaxes. The reason is simply that parallax itself is an effect of the satellite's motion around the Sun, manifested in the apparent displacement of a star's celestial position towards the Sun (Fig. 1). The displacement is

$$p = \pi R \sin\theta \tag{1}$$

where π is the parallax of the star, R the distance of the observer from the Sun expressed in astronomical units (AU), and θ the angle between the star and the Sun. For Gaia, R varies between 0.993 and 1.027, but for simplicity we ignore this variation in the following and use the mean value $R = 1.01$ (at L2).

To describe the geometry of the instrument with respect to the Sun we use notations as in Fig. 2. The spin relative to the Sun is given by the heliotropic spin phase Ω, which increases at an approximately constant rate of 60 arcsec s^{-1}.

In order to see how a bias in the parallax measurements could arise, imagine that all stellar parallaxes are artificially increased by a small amount $\delta\pi$ (the same for every star). Using simple geometry it is straightforward to derive the observable effects, in the astrometric fields, of such a shift. We then ask what particular variation of the instrument would completely 'absorb' the effects of $\delta\pi$. Then, if the instrument actually behaves in that way, without our knowing it, there would of course be no observed effects of the parallax shift and all derived parallaxes would be too small by the amount $\delta\pi$. In other words, that particular instrument variation would produce a global parallax bias of $\Delta\pi = -\delta\pi$. (Global, in this context, means that it is the same for all stars; cf. Appendix.)
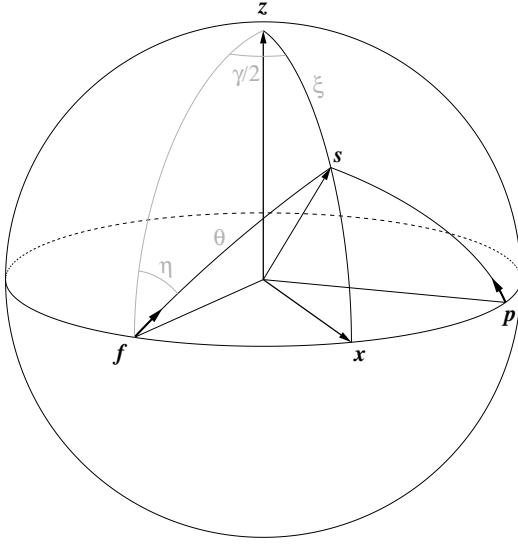
FIGURE 3: (Left) The short arrows at $\boldsymbol{p}$ and $\boldsymbol{f}$ show the observable effects of a (positive) parallax shift in the case when $\Omega = 0$.
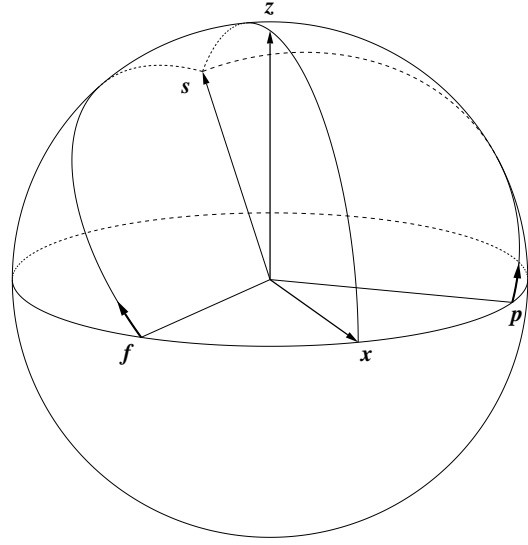
FIGURE 4: (Right) The observable effects of a (positive) parallax shift in the case when $\Omega = 180°$.

Figures 3 and 4 illustrate the observable effects of the parallax shift $\delta\pi$ in the two cases when $\Omega = 0$ and $180°$. As drawn in the figures, $\delta\pi > 0$. Let us first look at the case $\Omega = 0$ (Fig. 3). In the spherical triangle $\boldsymbol{fzs}$ we have

$$\left.\begin{array}{rcl} \sin\theta\sin\eta &=& \sin\xi\sin(\gamma/2) \\ \sin\theta\cos\eta &=& \cos\xi \end{array}\right\} \quad (\boldsymbol{f}\text{-field};\ \Omega = 0) \tag{2}$$

The along- and across-scan components of the parallactic displacement are

$$\left.\begin{array}{rclcl} p_\parallel &=& \delta\pi R\sin\theta\sin\eta &=& \delta\pi R\sin\xi\sin(\gamma/2) \\ p_\perp &=& \delta\pi R\sin\theta\cos\eta &=& \delta\pi R\cos\xi \end{array}\right\} \quad (\boldsymbol{f}\text{-field};\ \Omega = 0) \tag{3}$$

In the $\boldsymbol{p}$-field, the same formulae apply if $\gamma/2$ is replaced by $-\gamma/2$. It is seen that the effects of both displacements (in the $\boldsymbol{p}$ and $\boldsymbol{f}$ fields) can be completely absorbed by the following two changes: (i) the basic angle is *decreased* by the amount $2\delta\pi R\sin\xi\sin(\gamma/2)$, and (ii) the instrument is tilted an angle $-\delta\pi R\cos\xi/\cos(\gamma/2)$ about the $\boldsymbol{y}$ axis.[1] The second change is of course accommodated by the attitude determination.

Let us then consider the case $\Omega = 180°$ (Fig. 4). We find

$$\left.\begin{array}{rclcl} p_\parallel &=& -\delta\pi R\sin\theta\sin\eta &=& -\delta\pi R\sin\xi\sin(\gamma/2) \\ p_\perp &=& \delta\pi R\sin\theta\cos\eta &=& \delta\pi R\cos\xi \end{array}\right\} \quad (\boldsymbol{f}\text{-field};\ \Omega = 180°) \tag{4}$$

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[1] Strictly speaking, there are differential effects within each field of view that could *in principle* be used to disentangle a change in the basic angle from a parallax shift. However, the differential effects are about a factor 100 smaller than the total shifts (because the field size is of order 0.01 radian), and therefore in practice unobservable.

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while for the $\boldsymbol{p}$-field the sign of $\gamma/2$ should again be reversed. It is seen that the effect can be absorbed if (i) the basic angle is *increased* by $2\delta\pi R\sin\xi\sin(\gamma/2)$, and (ii) the instrument is tilted an angle $-\delta\pi R\cos\xi/\cos(\gamma/2)$ about the $\boldsymbol{y}$ axis.

Thus, for $\Omega = 0$ and $180°$ the along-scan effects of the parallax shift can be absorbed by a variation of γ by $\mp 2\delta\pi R\sin\xi\sin(\gamma/2)$. It is easy to show (cf. GAIA–LL–052) that these cases are just the extremes of a perfectly cosinusoidal variation with the heliotropic spin phase Ω. That is, a basic-angle variation according to

$$\Delta\gamma(\Omega) = -2\delta\pi R\sin\xi\sin(\gamma/2)\cos\Omega \tag{5}$$

would completely absorb the parallax shift $\delta\pi$ in all along-scan observations. Conversely, if the instrument experiences a variation according to

$$\gamma(\Omega) = \gamma_0 + a_1\cos\Omega \tag{6}$$

there will be a global parallax bias of

$$\Delta\pi = \frac{a_1}{2R\sin\xi\sin(\gamma/2)} \tag{7}$$

(since $\Delta\pi = -\delta\pi$), or $\Delta\pi \simeq 0.847\,a_1$ for $\xi = 50°$, $\gamma = 99.4°$, and $R = 1.01$ AU.[2]

A systematic basic-angle variation of the form (6) could be produced by the changing thermal impact of the solar radiation during the satellite spin, for instance if there is a slight asymmetry of the sunshield with respect to the spin axis. This clearly makes it necessary to put a separate requirement on the a_1 term, as opposed to general basic-angle fluctuations.

3 General basic-angle fluctuations

The basic angle (γ) is one of the geometrical calibration parameters routinely determined as part of the normal Gaia data analysis. Certain variations of the basic angle can also be determined by introducing the corresponding parameters, such as polynomial coefficients, as unknowns in the data analysis. Here I will briefly discuss the limitations of this procedure.

Let $B(f)$ be the one-sided power spectral density of $\gamma(t)$ (so that $\int_0^\infty B(f)\mathrm{d}t = \sigma_\gamma^2$ is the total variance of the basic-angle fluctuations). It can be assumed that $B(t)$ is continuous with superposed spikes e.g. at the harmonics $f_k = k/P$ of the spin period P ($= 6$ hr in the current design).

The geometrical basis for the determination of γ is the closure condition on each completed great-circle scan. Thus, in principle, we can obtain one independent determination of γ for each spin period. If there are no basic-angle variations at frequencies above the Nyquist frequency $f_{\mathrm{Ny}} = (2P)^{-1}$, then it follows from the sampling theorem that $\gamma(t)$ can be completely recovered from the measured data. Consequently, we only need to consider the

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[2]This is the same as Eq. (11) in GAIA–LL–052, in which however R was implicitly set to 1.

'high-frequency' part of $B(f)$, i.e., for $f > f_{\mathrm{Ny}}$. This includes all the harmonics f_k of the spin period.

According to the preceding analysis, a basic angle variation of the form (6) gives exactly the same observational results as a global shift of the parallaxes. Since the parallaxes are unknowns to be determined as part of the data analysis, it follows that a_1 cannot be introduced as an additional unknown, lest the problem becomes under-determined. Thus, $B(f_1)$ (or at least the cosine component of the variations at f_1) is unobservable.

Concerning the remaining part of $B(f)$ at $f > f_{\mathrm{Ny}}$, that is excluding f_1, limited experience with the Hipparcos data analysis suggests that also this can *in principle* be recovered in the data analysis, although only through combination of data from the whole mission. The accuracy and convergence properties of such a determination in the GIS scheme are however completely unknown. For this reason, the conservative assumption must be adopted, namely that such basic-angle fluctuations remain undetermined and thus contribute to the error budget. This applies to both the continuous and discrete components of the power spectrum.

In GAIA–LL–052 (Rev. 1) the error propagation from basic-angle fluctuations to parallax was considered in some detail, although only for the discrete frequencies f_k. It is possible to define, at least approximately, a filter function $K_\gamma(f)$ such that the rms contribution to the final parallax error is

$$\sigma_{\pi,\mathrm{ba}} = \left(\int_0^\infty B(f) K_\gamma(f) \mathrm{d}f \right)^{1/2} \tag{8}$$

We can assume that $K_\gamma(f) = 0$ for $f < f_{\mathrm{Ny}}$, as discussed above. The filter factor is different for each harmonic of the spin, depending on the value of γ. For example, with $\gamma = 99.4°$, variations at f_4 and f_7 are less damped than at f_1 and f_2 due to the approximate commensurabilities $4\gamma \sim 1$ rev and $7\gamma \sim 2$ rev. Actually, the function $K_\gamma(f)$ is not very well known as it depends on the detailed modelling of $\gamma(t)$ in the data analysis which is still undefined. However, since $B(f)$ will be steeply decreasing at high frequencies ($f > 1/\tau$, where the thermal time constant τ is of order several hours, TBC), it should be safe to assume that $K_\gamma(f) = K_\gamma(f_1)$ for $f > f_{\mathrm{Ny}}$. Based on numbers from GAIA–LL–052, and the fact that $\mathrm{COI}_\pi \propto N_{\mathrm{transit}}^{-1/2} (R \sin \xi)^{-1}$, we find

$$\sigma_{\pi,\mathrm{ba}} = \frac{(3.26/N_{\mathrm{transit}})^{1/2}}{2R \sin \xi \sin(\gamma/2)} \left(\int_{1/(2P)}^\infty B(f) \mathrm{d}f \right)^{1/2} \tag{9}$$

where, for the current (Gaia-2) design $N_{\mathrm{transit}} = 90$, $R = 1.01$, $\xi = 50°$, $\gamma = 99.4°$, and $P = 6$ hr, leading to a numerical factor 0.161 in front of the square-root of the integral.

4 Methods to constrain or eliminate a global parallax bias

Given that a_1 and a global parallax shift are observationally indistinguishable, there are basically three methods that can be used to constrain or eliminate such a global parallax bias: (i) constraining a_1 by design; (ii) measuring a_1 by dedicated hardware; or (iii) measuring $\Delta\pi$ by astrophysical modelling. Each method is briefly discussed below.

4.1 Constraining a_1 by design

According to (7), requiring an upper limit on $|a_1|$ is equivalent to a limit on $|\Delta\pi|$. As explained in the Appendix, different astrophysical applications have different requirements in terms of the parallax zero point. For some astrophysical problems a limit of $|\Delta\pi| < 0.1$ μas may be desirable. But a corresponding requirement on $|a_1|$ could probably never be guaranteed, at least within the current design paradigm of Gaia. It is therefore clear that the science goals cannot entirely rely on a stability requirement, but should be supplemented through the other methods described below. We may then ask what is the *maximum* limit on $|\Delta\pi|$ that would still make sense for (at least some of) the science goals. I would put this limit at 2 μas, corresponding to 10% of the currently predicted random error at 15th magnitude. At this bias level, the individual parallaxes would still be astrophysically useful and allow a lot of important science to be carried out, while it would clearly be detrimental for other applications such as the mean parallaxes of distant clusters.

4.2 Measuring a_1 by dedicated hardware

It is foreseen that Gaia will include a Basic Angle Monitoring (BAM) device, capable of monitoring changes in γ to a significantly higher precision than the general stability requirement. Such a device should clearly be capable to determine a_1, as well as other components of the (high frequency) variations, by proper spacing and precision of measurements. Note that the BAM device cannot, and need not, measure the absolute value of the basic angle, only its variation over a time scale of minutes to days.

To allow determination of $\gamma(t)$ at frequency f_1, the sampling frequency must be at least $2f_1$. However, this assumes that there are no fluctuations at higher frequencies that could contribute to the measured a_1 through aliasing. To guarantee 'clean' determination of a_1 in the presence of random fluctuations, a much higher sampling frequency is needed. The required oversampling factor depends on the bandwidth of the actual fluctuations, but a factor $N = 12$ (say) would appear safe. This means that the maximum interval between BAM readings should be $P/(2N) = 15$ min in the current design. If each reading is made with precision ϵ_γ (excluding bias term, assumed to be constant over at least a whole spin period), then all harmonic coefficients a_k, b_k of order $k = 1, 2, \ldots N$ can be determined with accuracy $\epsilon(a_i) = \epsilon(b_i) = \epsilon_\gamma N^{-1/2}$.

Thus, if 24 BAM readings at 0.5 μas precision are taken in every spin, then a_1 can be determined to about 0.15 μas per spin. This should be sufficient to guarantee that, after correction for the measured variations in the data analysis, the residual mission-wide a_1 term is less than 0.1 μas, corresponding to less than 0.1 μas global parallax bias.

4.3 Measuring $\Delta\pi$ by astrophysical modelling

A global parallax bias could be revealed by inconsistent parallaxes for objects at very great distances or with known distance ratios. In principle the procedure to determine $\Delta\pi$ is straightforward, but in reality it may be quite hard to achieve an accuracy of order 0.1 μas. Two examples are briefly considered here, mainly to illustrate the difficulties.

The simplest way would be to look at the measured parallaxes for objects that are distant enough to have negligible parallax. Quasars are the most obvious choice, because they are at cosmological distances (e.g., redshift $z > 0.01$ implies parallax < 0.03 μas), more or less pointlike, and relatively easy to distinguish from stars by their variability, colours, and absence of proper motion. They are, however, faint: of the $\sim 400\,000$ quasars that will be observed by Gaia down to 20th magnitude, 75% are fainter than 19th mag, 21% between 19th and 18th mag, and only 4% brighter than 18th mag. Potentially, they could jointly define the parallax zero point to an accuracy of about 0.2 μas. However, even a small fraction of contaminating stars could upset this determination. For instance, it can be estimated that there are some $100\,000$ faint stars, mostly in the Galactic halo, that have parallaxes and proper motions below 1.5 standard errors. Their mean parallax is around 100 μas. Thus, if only 1% of these stars were mistaken for quasars, they would bias the zero point determination by as much as 0.25 μas. Stellar contamination is insidious because it consistently works in the direction of increasing the mean parallax of the sample; by comparison, other problems intrinsic to the quasars, such as (variable) source structure, may increase the scatter but without biasing the mean observed parallax of the quasars.

An alternative way, which we may call the distance ratio method, uses photometrically estimated distance ratios to determine the parallax zero point. It is based on the assumption that stars with intrinsically identical properties (or samples with identical distribution of properties) can be identified at widely different distances. As an example, let us suppose that we observe a certain type of stars in the Milky Way, at a mean distance of 1 kpc and hence with measured parallax of order $\pi_{\mathrm{G}} \sim 1000$ μas, and that the same type can also be observed in the Magellanic Clouds at ~ 50 kpc or $\pi_{\mathrm{MC}} \sim 20$ μas. The apparent magnitude difference, after correction for interstellar extinction, provides an estimate of the true distance ratio X, which should equal the inverse parallax ratio after correction for the zero point error: $X = (\pi_{\mathrm{G}} - \Delta\pi)/(\pi_{\mathrm{MC}} - \Delta\pi)$. Given X, π_{G} and π_{MC} we can compute $\Delta\pi$. It can be seen that, in this particular example, an error in the photometrically determined distance ratio by 1% would give an error of 0.2 μas in $\Delta\pi$. Considering extinction and possible differences in chemical composition, such an accuracy may actually be difficult to achieve. On the other hand, there may be many independent such ways to estimate $\Delta\pi$.

In conclusion, astrophysical modelling is likely to determine the parallax zero point to an accuracy much better than 1 μas, and through a concerted effort perhaps even to 0.1 μas. The relevant scientific working groups should consider this problem.

5   Conclusion: Proposed requirements

A reasonable strategy will be to combine the three methods discussed above for maximum benefit: the stability requirement should guarantee an upper limit to $|\Delta\pi|$ that is acceptable for at least some of the science goals; the BAM should measure a_1 to sufficient accuracy to allow adequate correction of the data and for understanding the short-term behaviour of basic-angle variations; finally, several astrophysical methods should be used to check and determine the zero point independently.

Specifically, the following scientific requirements are proposed:

1. The short-term variation of the basic angle, as defined by the power spectral density integrated from $(2P)^{-1}$ to infinity, where P is the nominal spin period, shall be less than 7 μas rms.

2. The variation of the basic angle γ depending on the heliotropic spin phase Ω (defined as in Fig. 2) shall satisfy $\left|(1/\pi)\int_0^{2\pi}\gamma(\Omega)\cos\Omega\,\mathrm{d}\Omega\right| < 2$ μas.

3. Variations of the basic angle shall be monitored in flight. After suppression of frequencies below $(2P)^{-1}$, where P is the nominal spin period, the rms measurement error shall be less than 0.5 μas per 15 min interval.

Notes:

1. With current (Gaia-2) parameters, this requirement corresponds to a random contribution of $\sigma_{\pi,\mathrm{ba}} < 1.13$ μas to the end-of-mission standard error in parallax, according to Eq. (9).

2. By Fourier series analysis, $a_1 = (1/\pi)\int_0^{2\pi}\gamma(\Omega)\cos\Omega\,\mathrm{d}\Omega$, so the requirement is equivalent to $|a_1| < 2$ μas. With current (Gaia-2) parameters, it corresponds to $|\Delta\pi| < 1.69$ μas according to Eq. (7).

3. With current (Gaia-2) parameters, this requirement means that a_1 can be measured with standard error < 0.15 μas per spin period.

Appendix: Bias and the astrophysical use of parallax data

The primary scientific requirements for Gaia have been formulated in terms of the standard (rms) errors of the astrometric parameters, in particular the parallax.[3] The bias (or systematic error) is usually assumed to be small by comparison. However, it is important to somehow formulate a requirement also on the bias. This is quite difficult because biases may occur in a wide range of varieties, some of them more harmful than others, and also because the demarcation between 'systematic' and 'random' errors is to some extent arbitrary.

For example, instrument chromaticity may cause the parallax estimate for a given star (i) to be biased, depending on its colour. That is, if the observations were repeated many times under identical circumstances (but of course with different realizations of the photon noise and other truly random errors), the different parallax estimates resulting for this particular star would cluster around a value $\mathrm{E}[\hat{\pi}_i]$ that deviates from the true parallax $\bar{\pi}_i$. This is the abstract statistical meaning of bias, where the expectation operator E is interpreted as the ensemble average.

However, in the practical world Gaia will not be repeated (much less many times), and it makes more sense to define bias in terms of sample statistics. For instance, consider all stars (j) with approximately the same colour as i. Do we expect the individual parallax errors $\hat{\pi}_j - \bar{\pi}_j$ to cluster around a value significantly different from zero? Probably not, at least not due to chromaticity, because the different stars are observed under different geometrical conditions which tend to average out any systematic shifts in the measured along-scan coordinates, which is where the chromaticity occurs. Thus $\langle b_j \rangle \simeq 0$, where angular brackets denote the sample average. On the other hand, $\langle b_j^2 \rangle > 0$ and would therefore properly be included in the error budget. In other words, what is considered a systematic error (bias) for the individual star may be considered a random error (part of the standard error) for the sample. Similar considerations apply for other possible sources of systematic errors, e.g. from the geometrical calibration, CTI characteristics of the CCDs, or the attitude determination.

Using sample statistics to characterize an estimator immediately leads to the question: What constitutes a sample? From the astrophysical viewpoint this depends entirely on the scientific problem being addressed. Very often, we want to limit the sample to include only stars in a certain region of the sky, e.g., a stellar cluster or nearby galaxy. If the region is small enough, say less than a degree, it is very probable that $\langle b_j \rangle \neq 0$, since stars within a small area of the sky tend to be observed with similar scanning geometry. In this case we may want to consider $\langle b_j \rangle$ as a 'local systematic error' – but over how large area? – or we may think of it as spatially correlated errors and try to characterize them by means of a spatial correlation function.

The scientific importance of a possible parallax bias can be illustrated by a single example. The Large Magellanic Cloud (LMC) is one of the nearest and best-studied of the galaxies in the Local Group. Its distance (around 50 kpc, or 20 μas parallax) is an important step

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[3]The standard error is the square root of the estimated variance of a statistic. A statistic is a quantity computed from observed data, e.g. the estimated parallax ($\hat{\pi}$) of a star. With $\bar{\pi}$ denoting true parallax, we have the bias $b = \mathrm{E}[\hat{\pi}] - \bar{\pi}$, variance $\sigma^2 = \mathrm{E}[(\hat{\pi} - \mathrm{E}[\hat{\pi}])^2]$, and standard error $s = (\mathrm{E}[\sigma^2])^{1/2}$ of $\hat{\pi}$.

on the extragalactic distance scale. At present, the distance uncertainty is thought to be about 5%. Gaia will be able to observe of the order of 10^7 stars in the LMC, most of them faint ($G > 19$ mag). If the parallax of each star can be determined with a standard error of about 200 μas, then the potential precision of the *mean* parallax of the LMC is $200 \times 10^{-3.5} \simeq 0.06$ μas, corresponding to a relative precision of 0.3%. In reality the uncertainty of the mean parallax will be entirely determined by the level of systematic errors. For instance, if the local systematic error in the region of the LMC is (say) 1 μas, then the resulting relative distance uncertainty from Gaia will be 5%, and nothing is gained compared to the present knowledge. (Of course, Gaia would still contribute a lot to the distance scale, including the LMC distance, in many other ways.) Many similar examples can be given involving various open and globular clusters, dwarf galaxies, etc. There is thus a strong scientific interest to keep the parallax bias (however it is defined) well below 1 μas, and preferably at the 0.1 μas level.

While the precise meaning of parallax bias may vary according to the astrophysical context, the Gaia scientific requirements need to be formulated in terms of completely well-defined quantities. Since the Gaia data processing is not part of the responsibility of the industry, it is moreover necessary to specify how these quantities are to be derived from more fundamental data. The following two definitions may be useful: (i) The *global parallax bias* is $\Delta\pi \equiv \langle \hat{\pi}_j - \bar{\pi}_j \rangle$, where the average is taken over all the observed stars (j), irrespective of their colour and magnitude. (ii) The *chromatic parallax bias* is the $\sigma_{\pi,\mathrm{chr}}$ given by Eq. (10) in GAIA–LL–053 (8 May 2004). The requirements discussed in this note concern the global parallax bias.