Gaia astrometric, photometric, and radial-velocity performance assessment methodologies to be used by the industrial system-level teams

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Explanatory note This document has been compiled in the light of the Gaia ITT. The contents of this document have been (with a limited number of exceptions) literally copied into GAIA-EST-TN-00539, which is an official Reference Document in the Gaia ITT documentation package, with relevance notably for performance assessment-related scientific requirements in the Mission Requirements Document. Explanatory text contained in this document but lacking in GAIA-EST-TN-00539 has been typeset in red.

Abstract This document presents simple methodologies and formalisms, to be used by the Gaia industrial system-level teams, to calculate (i) the end-of-mission astrometric parallax standard error, systematic error, and the bright-star and calibration parallax noise floors; (ii) the end-of-mission photometric standard error; and (iii) the end-of-mission radial-velocity robust formal error.

1 Introduction

1.1 Astrometry

From the astrometric point of view, Gaia's scientific objective can be summarised as to observe the complete sample of all stars brighter than 20 mag with end-of-life astrometric standard errors (precisions) of 10 μas (yr^{-1}) at $V = 15$ mag (V denotes Johnson V magnitude). Systematic astrometric errors, after calibration, shall be below 1 μas (yr^{-1}) and the astrometric bright-star and calibration noise floors shall be kept, at the end of the mission, to a few μas (yr^{-1}).

1.2 Photometry

In order to be able to correct the astrometric measurements at the CCD level for unwanted systematic chromatic shifts, the global spectral energy distribution of *each* observed object must be measured, at the same spatial resolution and at the same epoch as the astrometric measurements, for example by means of say a handful of broad-band photometric filters covering the same wavelength range as the astrometric measurements. In the ESA Gaia-2 baseline design, the associated instrument is known as the Broad-Band Photometer (BBP). In addition to constituting the data needed for the chromatic correction task, the global spectral energy distribution of each observed object is also required to assist (or possibly even 'replace' under non-nominal sky conditions, e.g., in crowded fields) the astrophysical classification and characterisation of all observed objects, as discussed below.

In order to enable exploitation of Gaia's astrometric data and to meet the main scientific objective of the mission — to unravel the structure and chemical and dynamical evolution of the Galaxy — the astrophysical characteristics of *all* observed objects must be determined together with the astrometry, for example, by means of say a dozen medium-band photometric filters covering specific wavelength regions. In the ESA Gaia-2 baseline design, the associated instrument is known as the Medium-Band Photometer (MBP). The scientific interest lies in the classification and in-depth characterisation of all observed objects, i.e., in determining the object type (star, quasar, ...) and, for stars, the spectral type, luminosity class, effective temperature, luminosity, surface gravity, chemical composition, interstellar extinction, age, etc. (see technical notes 'Gaia scientific targets for PS design', UB-PWG-009, version 1.1, 14 May 2004 and 'Gaia scientific targets for PS design: quantification of priorities', UB-PWG-015, version 1.2, 14 May 2004).

The performance of the instruments described above will be refered to as Gaia's 'photometric performance'; the data that these 'photometric instruments' will deliver will be refered to as 'photometric data'.

1.3 Spectroscopy

Gaia's scientific objective, the study of the structure and chemical and dynamical evolution of the Galaxy, cannot be met without spectroscopic measurements being performed on-board, for the brightest subset of all observed objects, and sent to ground for scientific analysis. In the ESA Gaia-2 baseline design, the associated instrument is known as the Radial Velocity Spectrometer (RVS).

The spectroscopic measurements serve two main aims: (i) to provide, by means of Doppler-shift measurements, the missing, sixth phase-space coordinate (radial velocity), complementing the five 'astrometric' phase-space coordinates observed by Gaia (two position components, parallax, and two proper motion components), and (ii) to enable, by sophisticated spectroscopic analysis, (further) astrophysical characterisation of the observed objects. Whereas spectroscopy will sometimes complement/strengthen the photometric measurements (e.g., when determining effective temperatures or gravity-field strengths at stellar surfaces or the average 'metal' content of stellar atmospheres), spectroscopy will also provide unique diagnostical tools to study, e.g., stellar rotational velocities, the individual abundances of several chemical species (Ca, Si, Mg, ...) in stellar atmospheres, and the properties of interstellar matter.

1.4 Aim of this document

The main aim of this document is to present, in clear, unambiguous wording, top-level formalisms to calculate (i) end-of-mission astrometric standard errors and bright-star and calibration noise floors (Section 4); (ii) end-of-mission photometric standard errors (Section 5); and (iii) end-of-mission radial-velocity robust formal errors (Section 6), with agreed and common assumptions between all parties, ESA, Gaia Science Team, and industrial partners. The associated scientific performance requirements, defined in the 'Gaia Scientific Requirements' chapter of the 'Gaia Mission Requirements Document (MRD)', shall be understood in the context of this document. The formalisms shall, from now on, be used by both industrial system-level teams for all further performance assessment analysis and reporting.

Given the highly scientific nature of the techniques required for the astrophysical characterisation of objects by photometric and spectroscopic means, this document focuses on 'classical aperture photometry' and 'cross-correlation radial-velocity determination'. Specific scientific requirements needed for the proper derivation of astrophysical parameters from photometry and spectra are defined in the 'Gaia Scientific Requirements' chapter of the 'Gaia MRD' and are not discussed, or taken into account, here. Similarly, this document does not cover any time-dependent phenomena beyond a simple averaging over the end-of-mission number of CCD transits, whereas in reality, astrometric, photometric, and spectroscopic measurements of all objects will need to be made systematically and repeatedly throughout the operational lifetime, providing the crucial temporal sampling of positions, velocities, and intensities (magnitudes) to detect and characterise, e.g., stars in double and multiple systems, variable stars, planetary systems, and particular phases of stellar evolution. Moreover, all three formalisms (for astrometry, photometry, and radial velocity) assume to be dealing with a 'perfect star': isolated, unreddened, photometrically non-variable, with a well-behaved spectrum without extremities such as strong emission lines, without companions and/or planets, in a non-crowded field, without spectrum overlap, with a uniform, faint sky background, etc. The diversity of the real sky poses an extreme contrast with these highly simplified assumptions. As a result, the true *a posteriori* scientific data processing tasks which Gaia's scientists will be faced with will be massively more complex than the extremely simple formalisms outlined here. This also means that performance assessments calculated by means of the formalisms presented here are not (necessarily) representative of the true performance of Gaia, which will likely be degraded in practice. We therefore adopt an overall end-of-mission scientific contingency margin, not available to cover industrial/design aspects, of 20% ($m = 1.2$; see Section 2 for details).

2 Definition of quantities and symbols

Let us denote wavelength, expressed in units of nm, by λ. Let us define a photometric band j (for $j = 0, 1, 2, 3, \ldots$) by means of a normalised transmission profile $0 \leq B_j(\lambda) \leq 1$, defined over the wavelength interval $[\lambda_{\mathrm{min},j}, \lambda_{\mathrm{max},j}]$ and identical to zero ($B_j(\lambda) \equiv 0$) outside this interval. The object flux s_j (in units of photo-electrons, e^-) in photometric band j that can ideally (i.e., without losses due to possible windowing) be collected after a single CCD crossing of an object with a photon spectrum $N(\lambda)$ is then given by:

$$s_j\ [\mathrm{e}^-] = (D \times H) \cdot \tau_1 \cdot \int_{\lambda_{\mathrm{min},j}}^{\lambda_{\mathrm{max},j}} \mathrm{d}\lambda\ N(\lambda)\ T(\lambda)\ I(\lambda)\ B_j(\lambda)\ Q(\lambda), \tag{1}$$

where:

- $A \equiv D \times H$ denotes the pupil area in units of m^2 (D and H denote the along- and across-scan dimensions of the pupil, both expressed in units of m);
- τ_1 denotes the single-CCD integration time in units of s;
- $N(\lambda)$ denotes the star photon spectrum in units of photons $\mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{nm}^{-1}$, normalised according to the recipe described in Appendix D of technical note 'Stellar fluxes: transformations and calibrations', Gaia-JdB-005, 15 April 2003, revision 1;
- $T(\lambda)$ denotes the telescope transmission, including intermediate optics (when applicable);
- $I(\lambda)$ denotes the instrument transmission, including all optical elements;
- $B_j(\lambda)$ denotes the normalised transmission profile of photometric band j, excluding any effects related to the instrument transmission (or the telescope or the detectors). All transmission profiles $B_j(\lambda)$ thus have peak values of 1;
- $Q(\lambda)$ denotes the detector response (CCD quantum efficiency).

For each photometric band j, the object flux s_j can also be expressed in terms of a magnitude G_j (see also the 'Gaia Concept and Technology Study Report', ESA–SCI(2000)4, CTSR, page 239):

$$G_j\ [\mathrm{mag}] = -2.5 \cdot \log_{10}\left(\frac{s_j}{s_{j,\mathrm{A0V}}}\right), \tag{2}$$

where s_j is given by Equation (1) and the normalisation constant $s_{j,\mathrm{A0V}}$ refers to the value of s_j received from a zero-magnitude ($V = 0$ mag) unreddened A0V star, which, by definition, has $V = G_j = 0$ mag (V denotes Johnson V magnitude).

As an example, we can define the (design-dependent) Gaia G band and G magnitude (arbitrarily associated with photometric band $j = 0$) by setting $T(\lambda)$ = Astro telescope transmission, $I(\lambda) \equiv 1$ (on assumption that the astrometric measurements are made 'without instrument'), and $Q(\lambda)$ = Astro CCD quantum efficiency. Since the G magnitude is a broad-band, unfiltered, white-light magnitude, we set $B_0(\lambda) \equiv 1$; the associated wavelength interval $[\lambda_{\mathrm{min},0}, \lambda_{\mathrm{max},0}]$ is then set by the support of the product $T(\lambda)\ Q(\lambda)$ (in practice, $\lambda_{\mathrm{min},0} \sim 300$ nm and $\lambda_{\mathrm{max},0} \sim 1000$ nm). We then have:

$$s_0\ [\mathrm{e}^-] = (D \times H) \cdot \tau_1 \cdot \int_{\lambda_{\mathrm{min},0}}^{\lambda_{\mathrm{max},0}} \mathrm{d}\lambda\ N(\lambda)\ T(\lambda)\ Q(\lambda); \tag{3}$$

$$G_0\ [\mathrm{mag}] \equiv G = -2.5 \cdot \log_{10}\left(\frac{s_0}{s_{0,\mathrm{A0V}}}\right) = -2.5 \cdot \log_{10}\left(\frac{\int_{\lambda_{\mathrm{min},0}}^{\lambda_{\mathrm{max},0}} \mathrm{d}\lambda\ N(\lambda)\ T(\lambda)\ Q(\lambda)}{\int_{\lambda_{\mathrm{min},0}}^{\lambda_{\mathrm{max},0}} \mathrm{d}\lambda\ N_{\mathrm{A0V}}(\lambda)\ T(\lambda)\ Q(\lambda)}\right). \tag{4}$$

We define the following, general quantities and symbols:

- m denotes an overall end-of-mission scientific contingency margin. A value $m = 1.2$ (20% margin) shall be used in all performance assessments. As explained in Section 1.4, this margin is an ESA science margin, not meant for nor available to the industrial system-level teams. The scientific margin m is assumed to cover, among others:
  - 'scientific uncertainties' in the on-ground data analysis, including uncertainties related to relativistic corrections, aberration corrections, and the spacecraft and solar system ephemeris;
  - scientific effects such as the contribution to the astrometric error budget from the mismatch between the actual and the calibrating LSF, estimation errors in the sky background and total detection noise values that need to be fed to the centroiding algorithm, etc.;
  - the fact that the sky does not contain, as assumed in this document, 'perfect stars' but 'normal stars', which can be photometrically variable, have spectral peculiarities such as emission lines, have unrecognised companions, be located in crowded fields, etc.;
  - other astronomical environmental factors such as, e.g., localised enhanced sky background surface brightnesses, unrecognised small-scale sky background brightness gradients, unrecognised prompt particle events, etc.
- N_i is the number of instruments (viewing directions/telescopes) relevant for each measured quantity;
- $p_{\rm det}$ denotes the star detection and confirmation/cross-matching probability, which is a function of the on-board magnitude based on which the detection and confirmation/cross-matching is performed. This factor may be ignored in the end-of-mission performance budgets at all magnitudes, on assumption that $p_{\rm det} = 1$. Finite selection probabilities for stars in crowded areas, although highly relevant in reality, may be ignored in all three performance assessment methodologies;
- L denotes the nominal mission length, counted from the start of scientific observations (i.e., at the end of in-orbit calibration), expressed in units of s. A value $L = 5$ yr ($5 \cdot 365.25 \cdot 86\,400$ s) shall be used;
- $f_{\rm dead}$ is the fraction of the mission length for which scientific data will not be available during the on-ground processing. The ESA Gaia Project Team availability budget, from which $f_{\rm dead}$ follows, is detailed in Appendix A of GAIA-EST-TN-00539;
- Ω, expressed in units of steradians, denotes the effective solid angle per instrument of all CCDs in which relevant measurements are made. The word 'effective' in 'effective solid angle' refers to the light-sensitive CCD area exclusively: Ω thus excludes CCD dead zones in both along- and across-scan directions;
- $\tau = (1 - f_{\rm dead}) \cdot L \cdot \Omega / 4\pi$ is the sky-averaged total observing time available per object and per instrument, expressed in units of s;
- τ_1 denotes the single-CCD transit time, expressed in units of s;
- r, expressed in units of photo-electrons (e^-) per sample, denotes the total detection noise per sample (including, among others, detector read-out noise, detector dark noise, video chain noise, coupling and EMC noise, KTC noise, jitter noise, and coding [quantisation] noise);
- $N_{\rm line}$ denotes the number of TDI lines per CCD;
- $N_{\rm column}$ denotes the number of pixel columns per CCD;
- $N_{\rm strip}$ denotes the number of CCD strips, in which relevant measurements are made, in the focal plane;

- $N_{\rm row}$ denotes the number of CCD rows, in which relevant measurements are made, in the focal plane;
- $N_{\rm transit}$ denotes the sky-averaged total end-of-mission number of focal plane transits for all N_i telescopes combined;
- $N_{\rm eff} \equiv N_i \cdot \tau \cdot p_{\rm det} / \tau_1 = N_{\rm strip} \cdot N_{\rm transit}$ denotes the sky-averaged total number of detected-plus-confirmed CCD crossings, during which relevant measurements are made, of an object during the mission.

Note that many of the above quantities are instrument- and photometric-band-dependent: $N_{\rm eff}$, for example, may have a different numerical value for the astrometric measurements, the photometric measurements, and the spectroscopic (radial-velocity) measurements; $N_{\rm eff}$ may be different for different photometric bands as well.

3 Performance assessment ingredients: effects to be taken into account

The following effects, when applicable, shall be included in the performance assessments described in Section 4 (notably the calculation of the single-CCD transit centroiding error σ_ξ and the astrometric calibration error $\sigma_{\rm cal}$), in Section 5 (notably the calculation of the single-CCD transit photometric magnitude error $\sigma_{{\rm p},j}$ and the photometric calibration error $\sigma_{\rm cal}$), and in Section 6 (notably the calculation of the radial-velocity error $\sigma_{\rm r}$ and the spectroscopic calibration error $\sigma_{\rm cal}$). All effects listed below shall be included under the appropriate in-flight operating conditions (temperature, CCD operating mode, etc.). All error sources shall be included as random variables with typical/average/expected deviations (as opposed to best-case or worst-case deviations).

1. Sky background. A solar-type spectrum (unreddened G2V star) with a surface brightness $V = 22.5$ mag arcsec^{-2} shall be used (V denotes Johnson V magnitude). The sky background contribution shall be multiplied by a factor N_i in the case of the superposition of the N_i viewing directions (telescopes) in the focal plane;

2. Photon noise (from the source and from the sky background);

3. Total detection noise r (including, among others, detector read-out noise, detector dark noise, video chain noise, coupling and EMC noise, KTC noise, jitter noise, and coding [quantisation] noise);

4. Telescope transmission $T(\lambda)$, as function of wavelength, as function of location in the field of view, and as function of signal intensity;

5. Instrument transmission $I(\lambda)$, including intermediate optics, as function of wavelength, as function of location in the field of view, and as function of signal intensity;

6. Detector response $Q(\lambda)$ (CCD quantum efficiency), as function of wavelength, as function of pixel, and as function of signal intensity;

7. Detector sampling (windowing) and focal-plane geometry;

8. Overall effective polychromatic point spread function (PSF), as function of wavelength, as function of pixel, as function of location in the field of view, and as function of signal intensity, including:

   (a) a random, uniform intra-pixel centering ($\pm\frac{1}{2}$ pixel along-scan and $\pm\frac{1}{2}$ pixel across-scan) of the optical image in the window ('aperture'), modified as appropriate as a result of AOCS, detection, etc.;
   (b) optical diffraction;
   (c) aberrations;
   (d) optical vignetting;
   (e) straylight;
   (f) ghost images;
   (g) polarisation;
   (h) along- and across-scan source motion according to the nominal scanning law;
   (i) multi-phase TDI operation;
   (j) TDI rate errors;
   (k) attitude control errors;
   (l) line-of-sight motions and system tolerances;

(m) residual optical distortion;
(n) detector modulation transfer function (MTF), including integration MTF and charge diffusion MTF;
(o) charge-transfer inefficiency/efficiency (CTI/CTE);
(p) any aging effects of hardware components such as optics, mirror coatings, etc.;
(q) instrument tolerances (manufacture, assembly, integration, and launch);

9. CCD effects, including:
   (a) fringing;
   (b) radiation damage effects (both signal losses and random positional shifts due to charge trailing);
   (c) cosmetic defects (e.g., dark/bright pixels, dark/bright columns);
   (d) photo-response non-uniformity (PRNU);
   (e) saturation;
   (f) CCD and PEM/ADC non-linearity;
   (g) gain amplification;
10. Filter uniformity, homogeneity, light-leaks through the bandstop, and pre-launch calibration;
11. On-board processing, including:
   (a) master and/or slave clock instabilities (frequency drifts);
   (b) the finite resolution of the time tagging of the CCD sample data;
   (c) numerical binning;
   (d) CCD bias (dark noise) subtraction;
   (e) CCD column gain/response correction;
   (f) CCD cosmetic blemish reduction;
   (g) correction of scanning law;
   (h) correction of across-scan optical distortion;
   (i) correction of along-scan optical distortion;
   (j) detection and removal of prompt particle events;
   (k) clipping;
   (l) co-addition of CCD sample data/spectra;
   (m) compression.

4 Astrometric performance assessment methodology

4.1 Introduction

Since end-of-mission position, parallax, and proper motion standard errors, for a given star, are related in a straightforward way, there is no need to consider separate requirements or formalisms for position, parallax, and proper motion standard errors. We therefore limit the discussion in this document to parallaxes exclusively.

4.2 End-of-mission parallax standard error

The sky-averaged, end-of-mission parallax standard error σ_π, expressed in units of μas, shall be calculated as (see also the 'Gaia Concept and Technology Study Report', ESA–SCI(2000)4, CTSR, page 261):

$$\sigma_\pi \ [\mu\text{as}] = m \cdot g_\pi \cdot \sqrt{\frac{\sigma_\xi^2 + \sigma_{\text{cal}}^2}{N_{\text{eff}}}}, \tag{5}$$

where:

- g_π denotes the dimensionless geometrical parallax factor which relates the scanning geometry to the determination of the astrometric parameters. The quantity g_π varies as function of position in the sky, mainly as function of ecliptic latitude, but a sky-averaged value shall be used. For a given scanning law, the sky-average value of g_π depends mainly on the solar aspect angle ξ. (Reversely, however, the solar aspect angle ξ not only affects the geometrical parallax factor g_π but also, e.g., the uniformity of the sky sampling, the speed of the spin axis on the sky and thus the spin-axis precession period, the across-scan motion of objects in the focal plane, etc.) The simplified parametrisation $g_\pi = 1.47/\sin\xi$ shall be used (corresponding to $g_\pi \simeq 1.92$ for $\xi = 50°$);
- σ_ξ^2, expressed in units of μas^2, denotes the variance of the line spread function (LSF) centroiding for a single-CCD transit. The error σ_ξ shall be evaluated with dedicated Monte Carlo centroiding simulations, explicitly taking all effects described in Section 3 into account. The error σ_ξ may also be evaluated with the theoretical Cramér–Rao bound, provided that supporting evidence is presented demonstrating that (i) the sub-optimality of a practical centroiding algorithm can be neglected; and (ii) the sampling of the LSF is adequate, for all stellar types and at all wavelengths.

  The quantity σ_ξ denotes the random part of the single-CCD transit centroiding error. Any bias in the astrometric centroid location as a result of sub-pixel location of the optical image shall either explicitly be shown to be negligible or be included in σ_ξ^2;
- σ_{cal}, expressed in units of μas, is a calibration term, which is discussed further in Section 4.3.

Explanatory note:

- Inferior-quality astrometric data from the (ASM) star-mapper detection CCDs shall be ignored as contributor to the end-of-mission astrometric standard error.

4.3 Bright-star and calibration parallax noise floors

In the on-ground data processing, the location (centroiding) measurements in pixel coordinates need to be transformed to angular field coordinates through a geometrical calibration of the astrometric focal plane, and subsequently to coordinates on the sky through calibrations of the astrometric instrument attitude and basic angle. The random (residual) errors associated with these transformations are represented by the variance σ_{cal}^2.

In the absence of centroiding errors ($\sigma_\xi = 0$), Equation (5) reduces to:

$$\sigma_{\pi,\mathrm{cal}} = m \cdot g_\pi \cdot \sqrt{\frac{\sigma_{\mathrm{cal}}^2}{N_{\mathrm{eff}}}}, \tag{6}$$

where the quantity $\sigma_{\pi,\mathrm{cal}}$ is defined as the end-of-mission *calibration* parallax noise floor. The end-of-mission *bright-star* parallax noise floor $\sigma_{\pi,\mathrm{min}}$, on the other hand, is composed of two terms (centroiding *and* calibration errors), and is defined as the end-of-mission parallax standard error (Equation 5) reached for bright stars (e.g., $V < 10$ mag):

$$\sigma_{\pi,\mathrm{min}} = m \cdot g_\pi \cdot \sqrt{\frac{\sigma_{\xi,\mathrm{min}}^2 + \sigma_{\mathrm{cal}}^2}{N_{\mathrm{eff}}}}, \tag{7}$$

where $\sigma_{\xi,\mathrm{min}}$ denotes the single-CCD centroiding error σ_ξ that is reached for bright stars. Note that the 'Gaia MRD' bright-star requirement is a *bright-star*, and not a *calibration*, parallax noise floor requirement. Note also that neither the 'Gaia MRD' nor this document specifies how the maximum-allowed parallax error for bright stars shall be partitioned between centroiding errors and calibration errors.

As explained above, the quantity σ_{cal} represents astrometric telescope/instrument stability and random calibration errors, and includes:

1. Residual random errors on the chromaticity calibration (quantified by $\sigma_{\mathrm{chromaticity}}$). It shall be demonstrated (in consultation with the Gaia Science Team) that appropriate (broad-band photometric) measurements will be available for each observed object to allow a chromaticity correction with a random error $\leq$1%, such that:

$$\sigma_{\mathrm{chromaticity}} = 0.01 \cdot \sqrt{\left([\langle \Delta u \rangle_{\mathrm{AL}}]_{\mathrm{RMS}}\right)^2 + \left(\frac{[\Delta u]_{\mathrm{D}}}{4}\right)^2}, \tag{8}$$

   where:

   - Δu denotes the (along-scan; AL) chromatic displacement between an unreddened B1V star and an unreddened M6V star;
   - The AL chromatic displacement for a pair of stars with different spectral types is defined as the difference between the monochromatic LSF centroids at the effective wavelengths λ_{eff} of the two stars (see also Equation 3 in technical note 'Chromaticity specification', Gaia-LL-053, 8 May 2004, V.1);
   - The centroid of the LSF shall be defined, for this purpose exclusively, as the symmetry point of a Gaussian function least-squares fitted to the LSF, in which the Gaussian has the same full-width at half-maximum as the LSF;
   - The effective wavelength λ_{eff} of a star is defined by (see also Equation 2 in Gaia-LL-053):

$$\lambda_{\mathrm{eff}} = \frac{\displaystyle\int_{\lambda_{\mathrm{min},0}}^{\lambda_{\mathrm{max},0}} \mathrm{d}\lambda\ N(\lambda)\ T(\lambda)\ Q(\lambda)}{\displaystyle\int_{\lambda_{\mathrm{min},0}}^{\lambda_{\mathrm{max},0}} \mathrm{d}\lambda\ \lambda^{-1}\ N(\lambda)\ T(\lambda)\ Q(\lambda)}; \tag{9}$$

   - $\langle \Delta u \rangle_{\mathrm{AL}}$ denotes, for a given across-scan coordinate, the AL-averaged value of Δu (as in Equation 5 in Gaia-LL-053);
   - $[\langle \Delta u \rangle_{\mathrm{AL}}]_{\mathrm{RMS}}$ denotes the RMS AL-averaged chromaticity (as in Equation 7 in Gaia-LL-053);

- $[\Delta u]_\mathrm{D}$ denotes the total, field-averaged chromaticity dispersion (as in Equation 8 in Gaia-LL-053);

2. Residual random errors on the calibration of the geometrical transformation from astrometric focal plane (pixel) coordinates to field coordinates, including any uncalibrated systematic shifts caused by, e.g., CCD/CTI effects, chromaticity, etc. (quantified by $\sigma_\mathrm{geometrical}$). It shall be assumed that:
$$\sigma^2_\mathrm{geometrical} = \sigma^2_\mathrm{geometrical,design} + \sigma^2_\mathrm{geometrical,cal}, \tag{10}$$
where:
$$\sigma_\mathrm{geometrical,cal} = \sqrt{\frac{1}{2}} \cdot \frac{1\ \mu\mathrm{as} \cdot \sqrt{N_\mathrm{transit}}}{g_\pi}; \tag{11}$$

3. Random errors in the calibration model of the satellite attitude (quantified by σ_attitude). It shall be assumed that:
$$\sigma^2_\mathrm{attitude} = \sigma^2_\mathrm{attitude,HFAD} + \sigma^2_\mathrm{attitude,OGAD}, \tag{12}$$
where, on assumption that a sufficient number of bright stars is observed at each instant of time with sufficiently accurate single-CCD centroiding standard errors σ_ξ to allow appropriate on-ground attitude reconstruction, we have:
$$\sigma_\mathrm{attitude,OGAD} = \sqrt{\frac{1}{2}} \cdot \frac{1\ \mu\mathrm{as} \cdot \sqrt{N_\mathrm{transit}}}{g_\pi}; \tag{13}$$

4. Residual random astrometric telescope/instrument errors (quantified by $\sigma_\mathrm{telescope}$), after basic-angle calibration, taking into account the thermo-mechanical stability of the astrometric telescope, the thermo-mechanical stability of the astrometric focal plane, and metrology errors associated with the basic-angle monitoring.

Each item in the above list shall be considered as corresponding to a single focal-plane passage (field-of-view, or FoV, transit). Since σ_cal in Equation (5) applies at CCD level, an equivalent specification at FoV transit level must be $\sqrt{N_\mathrm{strip}}$ times stricter (smaller), so that:
$$\sigma^2_\mathrm{cal} = N_\mathrm{strip} \cdot \left\{\sigma^2_\mathrm{chromaticity} + \sigma^2_\mathrm{geometrical} + \sigma^2_\mathrm{attitude} + N_i \cdot \sigma^2_\mathrm{telescope}\right\}, \tag{14}$$
or:
$$\frac{\sigma^2_\mathrm{cal}}{N_\mathrm{eff}} = \frac{1}{N_\mathrm{transit}} \cdot \left\{\sigma^2_\mathrm{chromaticity} + \sigma^2_\mathrm{geometrical} + \sigma^2_\mathrm{attitude} + N_i \cdot \sigma^2_\mathrm{telescope}\right\}. \tag{15}$$
Further to the above list, there are several ‘scientific calibration effects’ (e.g., relativistic aberration corrections), all of which will be taken into account in the on-ground data processing. These effects shall not be explicitly included in σ_cal and are assumed to be covered by the 20% ESA science margin.

4.4 Systematic astrometric (parallax) error

The quantity σ_cal, described in Section 4.3, describes a random astrometric error related to astrometric telescope/instrument stability and calibrations. Some of the effects discussed in Section 4.3 can, in principle, also induce unwanted systematic astrometric errors. Let us denote $\sigma_{\mathrm{sys},i}$ as the end-of-mission systematic (parallax) error induced by transformation/calibration/effect i (i can denote, e.g., calibration of the basic angle, geometrical calibration of the astrometric focal plane, chromaticity, CCD/CTI effects, etc.). All identified systematic sources of error $\sigma_{\mathrm{sys},i}$ in any adopted design solution shall be demonstrated to be insignificant in the end-of-mission data products (i.e., positions, parallaxes, and proper motions):
$$\sigma_\mathrm{sys} = \sqrt{\sum_i \sigma^2_{\mathrm{sys},i}} \leq \sigma_\mathrm{sys,max}, \tag{16}$$
where $\sigma_\mathrm{sys,max}$ is the maximum tolerable (overall) systematic error as defined in scientific requirement SCI-260 in the ‘Gaia MRD’, and the sum is over all identified systematic sources of error.

5 Photometric performance assessment methodology

5.1 Introduction

In the on-ground photometric data processing for the ESA Gaia-2 baseline design, the object fluxes s_j and sky background intensities are currently foreseen to be solved for by a maximum likelihood PSF-fitting technique in which the object positions are assumed to be known with sufficient precision from the Astro measurements. In this process, the positions of (faint) contaminating stars will be required, and assumed to be known from the analysis of, among others, star-mapper data. Since (i) performance estimates for PSF-fitting techniques are not amenable to be cast into a simplified recipe, and (ii) the principles and procedures used in the *a posteriori* ground-based photometric data processing will depend on the characteristics of the photometric data (which, in turn, depend on the design solution/implementation of the instruments), the top-level methodology for photometric performance assessments set forward in this document is based on a simplistic, canonical 'aperture photometry' approach. This document assumes that the photometric instruments deliver data from which the object fluxes s_j in a number of specific photometric bands j can be extracted. It is furthermore assumed here that these fluxes can be used to determine the relevant quantities, such as interstellar extinctions (see Section 1), needed for the scientific exploitation of the astrometric data.

Each photometric band j is defined by means of a normalised transmission profile $0 \leq B_j(\lambda) \leq 1$, defined over the wavelength interval $[\lambda_{\mathrm{min},j}, \lambda_{\mathrm{max},j}]$ (photometric bands are often refered to with their central wavelength, denoted λ_0). The current ESA Gaia-2 baseline design considers 5 broad photometric bands (the so-called 'C1B baseline') and 13 medium photometric bands (the so-called 'C1M baseline', excluding the C1M326 band).

5.2 End-of-mission photometric performance

The sky-averaged, end-of-mission photometric magnitude error $\sigma_{G,j}$ for photometric band j, expressed in units of mag, shall be calculated as (see also the 'Gaia Concept and Technology Study Report', ESA–SCI(2000)4, CTSR, Section 7.5.1, pages 262–263):

$$\sigma_{G,j}\ [\mathrm{mag}] = m \cdot \sqrt{\frac{\sigma_{\mathrm{p},j}^2 + \sigma_{\mathrm{cal}}^2}{N_{\mathrm{eff}}}}, \tag{17}$$

where:

- The single-CCD transit photometric magnitude error $\sigma_{\mathrm{p},j}$, expressed in units of mag, is defined as:

$$\sigma_{\mathrm{p},j}\ [\mathrm{mag}] = 2.5 \cdot \log_{10}(e) \cdot \frac{\sqrt{f_{\mathrm{aperture}} \cdot s_j + (b_j + r^2) \cdot n_{\mathrm{s}} \cdot \left(1 + \frac{n_{\mathrm{s}}}{n_{\mathrm{b}}}\right)}}{f_{\mathrm{aperture}} \cdot s_j}; \tag{18}$$

- $2.5 \cdot \log_{10}(e) = 1.086\ldots$ is a constant ($e = 2.718\ldots$);
- We assume, following an 'aperture photometry' approach, that the object flux s_j is measured in a rectangular 'aperture' (window) of n_{s} along-scan object samples. A sample itself may be composed of several pixels (numerically) binned across scan. The object and background windows (see below) shall be defined such as to contain the object flux and sky background in an optimum manner according to object magnitude, whilst satisfying telemetry constraints, imaging requirements, bright-star sample modes, and calibration modes;
- The sky background b_j (see below) is assumed to be measured in n_{b} background samples. These samples are identical in size to and spatially distinct from the object samples. The contribution from the star in the background samples shall be shown to be negligible;

- $f_{\mathrm{aperture}} \cdot s_j$, expressed in units of photo-electrons (e^-), denotes the object flux in photometric band j contained in the 'aperture' (window) of n_{s} samples, after a CCD crossing. The quantity s_j shall be evaluated by means of Equation (1). Any light loss — along-scan and/or across-scan — due to radiation damage, vignetting, and/or the finite extent of the 'aperture' of n_{s} samples shall be taken into account by an appropriate reduction of s_j through multiplication with the fraction $0 \leq f_{\mathrm{aperture}} \leq 1$. This quantity, for a given 'aperture' size, is a function of the precise shape of the effective PSF, which, in turn, depends on the position in the field of view, the stellar spectrum, etc.;

- b_j, expressed in units of photo-electrons (e^-) per sample, denotes the sky-background level in an object sample of photometric band j. The sky background contribution shall be multiplied by a factor N_i in the case of the superposition of the N_i instruments in the focal plane;

- σ_{cal}, expressed in units of mag, is a calibration term, which is discussed further in Section 5.3.

In the above methodology, all effects described in Section 3 shall explicitly be taken into account

5.3 Photometric calibration error

In the on-ground data processing, spatial and temporal changes of the response of the photometric instruments need to be calibrated (see, for example, pages 263–264 in the 'Gaia Concept and Technology Study Report', ESA–SCI(2000)4, CTSR, Section 7.5.2 and Appendix A in 'Algorithms for GDAAS Phase II — Definition', L. Lindegren, 10 July 2003, Gaia-LL-044). The random (residual) errors associated with these calibrations are represented by the quantity σ_{cal}.

A value $\sigma_{\mathrm{cal}} = 0.030$ mag (30 mmag) shall be used. This number represents a fiducial reference value, chosen to enable meaningful comparisons of different designs, and bears no direct physical significance.

6 Radial-velocity performance assessment methodology

6.1 Introduction

For faint stars, the spectroscopic signal collected per CCD or FoV crossing is generally not large enough to extract reliable epoch radial velocities at CCD-transit or FoV-transit level. Performance estimates for single-CCD or single-FoV transit spectra can thus not simply be scaled to end-of-mission estimates by means of the sky-averaged total number of detected-plus-confirmed CCD or FoV crossings of an object during the mission, like for astrometric and photometric performance estimates (Equations 5 and 17). Instead, it shall be assumed for all stars, for this purpose exclusively, that one single end-of-mission composite spectrum will first be reconstructed by proper co-addition of all spectra collected during all $N_{\rm eff}$ CCD crossings throughout the mission lifetime (Appendix A). A single mission-averaged radial velocity will then be extracted from this end-of-mission composite spectrum. All spectroscopic scientific performance requirements listed in the 'Gaia Scientific Requirements' chapter of the 'Gaia MRD' refer to this assumed procedure, although it is currently foreseen in the *a posteriori* on-ground data analysis to actually derive single-FoV transit spectra, and to extract associated epoch radial velocities, whenever this proves possible in practice.

6.2 End-of-mission radial-velocity robust formal error

The end-of-mission radial-velocity robust formal error $\sigma_{\rm Vrad}$, expressed in units of km s^{-1}, shall be calculated as:

$$\sigma_{\rm Vrad}\ [\text{km s}^{-1}] = m \cdot \sqrt{\sigma_{\rm r}^2 + \frac{\sigma_{\rm cal}^2}{N_{\rm transit}}}, \tag{19}$$

where:

- $\sigma_{\rm r}^2$, expressed in units of km^2 s^{-2}, denotes the variance of the end-of-mission radial-velocity error distribution. The radial-velocity error distribution shall be derived, via dedicated Monte Carlo simulations in which all effects described in Section 3 are explicitly taken into account, by cross-correlation of simulated spectra with an appropriate, high-quality synthetic template spectrum (Appendix B).

  The radial-velocity error distribution resulting from the Monte Carlo experiments may exhibit a small fraction of outliers due to secondary (incorrect) cross-correlation peaks. Therefore, the variance $\sigma_{\rm r}^2$ of the end-of-mission radial-velocity error distribution shall be calculated by means of a robust estimator, as:

  $$\sigma_{\rm r}^2 = \frac{(\delta_{\rm r(0.8415)} - \delta_{\rm r(0.1585)})^2}{4}, \tag{20}$$

  where $\delta_{\rm r(0.8415)}$ and $\delta_{\rm r(0.1585)}$ denote, respectively, the 84.15$^{\rm th}$ and 15.85$^{\rm th}$ percentiles of the numerically sorted distribution $\delta_{\rm r(...)}$ of radial-velocity errors $\delta_{\rm r}$. For a particular Monte Carlo realisation, the radial-velocity error $\delta_{\rm r}$ is defined as $\delta_{\rm r} = {\rm Vrad}_{\rm est} - {\rm Vrad}_{\rm true}$, where ${\rm Vrad}_{\rm true}$ denotes the true radial velocity (used as nominal value in the simulations) and ${\rm Vrad}_{\rm est}$ denotes the estimated radial velocity (derived by cross-correlating the simulated spectrum with the synthetic template spectrum).

  Similarly, the expectation of the radial-velocity error distribution, associated with the systematic error, shall be derived using the median ($\delta_{\rm r(0.5000)}$) of the sorted distribution of radial-velocity errors $\delta_{\rm r}$. All identified systematic sources of error in any adopted design solution, e.g., from CCD/CTI effects, shall be demonstrated to be insignificant, calibratable, or consistent with the overall allocation to $\sigma_{\rm cal}$ (see below). Any bias of the radial-velocity error distribution as a result of sub-pixel location of the spectrum shall either explicitly be shown to be negligible or be included in $\sigma_{\rm r}^2$;

- $\sigma_{\rm cal}$, expressed in units of km s^{-1}, is a calibration term, which is discussed further in Section 6.3.

6.3 Radial-velocity calibration error

In the on-ground data processing, the pixel coordinates shall be converted to wavelengths through a 'wavelength calibration procedure', involving, among others, a geometrical calibration of the radial-velocity focal plane and calibrations of the radial-velocity instrument attitude. The random (residual) errors associated with these transformations are represented by the variance $\sigma^2_{\rm cal}$. The quantity $\sigma_{\rm cal}$ thus includes:

1. Residual random errors on the calibration of the geometrical transformation from radial-velocity focal plane (pixel) coordinates to field coordinates, including any uncalibrated systematic shifts caused by, e.g., CCD/CTI effects, etc. (quantified by $\sigma_{\rm geometrical}$);

2. Random errors in the calibration model of the satellite attitude (quantified by $\sigma_{\rm attitude}$);

3. Residual random radial-velocity telescope/instrument errors (quantified by $\sigma_{\rm telescope}$), after calibration, taking into account the thermo-mechanical stability of the radial-velocity telescope, the thermo-mechanical stability of the radial-velocity instrument optics, including intermediate optics, and the thermo-mechanical stability of the radial-velocity focal plane.

Each item in the above list shall be considered as corresponding to a single focal-plane passage (FoV transit), consistent with the appearance of $\sigma_{\rm cal}$ in Equation (19):

$$\sigma^2_{\rm cal} = \sigma^2_{\rm geometrical} + \sigma^2_{\rm attitude} + \sigma^2_{\rm telescope}. \tag{21}$$

The above-defined quantities are not (necessarily) identical to the identically-named quantities defined in Section 4.3.

Further to the above list, there are several 'scientific calibration effects' (e.g., template-mismatch errors, residual random errors in the derivation of the locations of the centroids of the reference spectral lines used for the wavelength calibration, etc.), all of which will be taken into account in the on-ground data processing. These effects shall not be explicitly included in $\sigma_{\rm cal}$ and are assumed to be covered by the 20% ESA science margin.

Appendix A: Guidelines for the construction of a single, end-of-mission composite spectrum

The following steps shall be taken:

1. All individual CCD spectra should be brought on a common 'pixel scale' (gain-bias scale) by correcting for pixel offsets, pixel-to-pixel responses, etc. Cosmic rays should be removed and proper corrections for CCD blemishes shall be made.

2. All individual CCD spectra should be resampled to a common, reference 'sampling framework', which can be assumed to be defined, arbitrarily but without loss of generality, by the first observed spectrum. Proper corrections, for each observed spectrum, shall be made for:

   - scanning-law effects;
   - the 'random', two-dimensional positioning of each spectrum with respect to the CCD pixels;
   - differential dispersion-law changes over the field of view of the spectrograph; and
   - dispersion-law changes with time.

   These corrections shall bring all spectra on a common, reference sample-wavelength scale, i.e., all corresponding samples in all spectra will have the same properties (e.g., central wavelengths, spectral widths, etc.) as the associated samples in the reference spectrum.

3. All individual spectra shall finally be co-added, sample by sample, to produce the required end-of-mission composite spectrum.

Appendix B: Cross correlation

The radial velocity of a star shifts the wavelength of a given photon emitted by the star according to the Doppler-shift formula:

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{star}}}{\lambda_{\text{star}}} = \frac{\text{Vrad}}{c}, \tag{22}$$

where:

- λ_{star} is the wavelength of a given photon, emitted by the star, in the reference frame of the star;
- λ_{obs} is the wavelength of the same photon in the reference frame of the observer;
- Vrad is the velocity of the star, projected along the line of sight from the observer to the star, in the reference frame of the observer;
- c is the velocity of light.

The wavelengths of the photons emitted by the star are all shifted according to Equation (22). Therefore, the full spectrum of the star is 'shifted' toward blue wavelengths if the star moves toward the observer (Vrad < 0) and toward red wavelengths if the star moves away from the observer (Vrad > 0).

The method that shall be used to derive the radial velocity is the cross correlation of the spectrum of the star of unknown radial velocity with a reference spectrum with a known radial velocity (typically 0 km s^{-1}). The principle of the cross-correlation method is to shift the reference spectrum in radial velocity, step by step, and to evaluate, at each step, 'how well' the lines of the reference spectrum coincide with the lines of the spectrum that is being analysed. The degree of coincidence of the lines of the two spectra is evaluated by the correlation coefficient of the two spectra (Equation 23). The correlation coefficient is small when the two spectra are broadly separated in radial velocity and is maximal when the two spectra are 'in phase', i.e., when the centroids of all spectral lines of the two spectra coincide.

Once the correlation coefficients have been calculated for a broad range of radial-velocity shifts of the reference spectrum, one obtains a distribution of pairs (radial-velocity shift, correlation coefficient). This distribution has a peak-like shape and is often referred to as the '(cross-)correlation peak' or '(cross-)correlation function'. The radial velocity of the target star is derived by finding the radial-velocity shift corresponding to the maximum of the correlation function.

The main steps of the derivation of the radial velocity by cross correlation with a reference spectrum are:

1. Select a reference spectrum with stellar parameters (effective temperature, surface gravity, metallicity, projected rotational velocity, etc.) and a spectral resolution similar to those of the spectrum that is being analysed. For the performance assessments, the 'true' star spectrum (noise-free) shall be used.
2. Shift the reference spectrum in radial velocity, step by step, to cover the radial-velocity range from -1000 to $+1000$ km s^{-1}.
3. At each discrete radial-velocity step:
   (a) Sample the reference spectrum with the same sampling as the spectrum that is being analysed.
   (b) Compute the correlation coefficient of the reference spectrum and the spectrum that is being analysed:

$$r(\text{Vrad}) = \frac{\sum_{i=1}^{N} (F(i) - \langle F \rangle) \cdot (F_r(i) - \langle F_r \rangle)}{\sqrt{\sum_{i=1}^{N} (F(i) - \langle F \rangle)^2} \cdot \sqrt{\sum_{i=1}^{N} (F_r(i) - \langle F_r \rangle)^2}}, \tag{23}$$

where:

- $r(\text{Vrad})$ is the correlation coefficient of the spectrum that is being analysed with respect to the reference spectrum shifted by Vrad km s^{-1};
- N is the number of samples per spectrum;
- $F(i)$ is the number of photo-electrons recorded in the i^{th} sample of the spectrum that is being analysed;
- $\langle F \rangle$ is the mean number of photo-electrons (averaged over the N samples) recorded for the spectrum that is being analysed;
- $F_r(i)$ is the number of photo-electrons recorded in the i^{th} sample of the shifted reference spectrum;
- $\langle F_r \rangle$ is the mean number of photo-electrons (averaged over the N samples) recorded for the shifted reference spectrum.

4. Derive the radial velocity of the target star by determining the radial-velocity shift of the reference spectrum corresponding to the maximum of the correlation function. To achieve a sub-step resolution, a classical technique is to fit the top of the correlation function with a low-order polynomial (e.g., a parabola).

Note: to be properly compared, the two spectra should have the same slopes of their continua.